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No. 858.

WHEEL CONCENTRATIONS AND FATIGUE FORMULAS IN BRIDGE DESIGN.

An Informal Discussion at the Annual Convention, June 27th, 1899.

SUBJECTS FOR DISCUSSION:

"Should the use of the Method of Wheel Concentrations be discontinued in determining the Stresses in Railroad Bridges?"

"In view of present knowledge of the Effect of Repeated Applications of Load, should Fatigue Formulas be used in Bridge Design?"

By Messrs. GUSTAV LINDENTHAL, PALMER C. RICKETTS, W. A. DOANE, GEORGE S. MORISON, W. L. COWLES, E. M. SCOFIELD, HENRY B. SEAMAN, J. P. SNOW, EDWIN THACHER, J. A. L. WADDELL, O. E. SELBY, HENRY S. PRICHARD, CHARLES B. WING and EDGAR MARBURG.

GUSTAV LINDENTHAL, M. Am. Soc. C. E.—The objection to the method of wheel concentrations for finding the strains in bridge members does not arise so much from the greater labor of calculation, but because the assumption of wheel loads as resting on knife-edges is a false and unnecessary refinement, for, in any case, the rails and ties intervening between the wheels and the bridge structure distribute the concentrated loads upon the length of the wheel base. It may be correctly assumed, for the bridge structure, that the loads rest on skids, instead of on wheels; the rails representing the skids. The strains in the floor and in the trusses are thus more accurately found, for the train load plus the excess load of the locomotive, than by the method of wheel concentrations, which should be discontinued. Mr. Lindenthal.

Mr. Lindenthal.

There are indications that the growth of weight in locomotives will soon come to an end; it will be determined, not by the strength of rails, bridges or drawbar heads, which can be increased at will, but by the hardness of steel, in wheel tires and rails, necessary to resist the wear and tear of the heavy engines. It will reach the limit when steel in the railhead would be crushed, or, if made still harder, would be too brittle for safety. It may be fairly assumed that 120 tons on a wheel base of 15 ft. is the practicable and economic limit of weight concentration on railroad track. This represents a quiescent load of 16 000 lbs. per lineal foot, corresponding to 30 000 lbs. per wheel; or from 60 000 to 75 000 lbs. pressure per square inch on the wheel tire and rail, which is close to, or exceeding, the elastic limit of hard steel.

Some rails have, indeed, been made of hard nickel steel, which may safely resist greater pressures, but they cost about double as much as ordinary rails. They are used only for the outside of curves, and, therefore, do not determine ordinary service conditions.

Fatigue formulas in bridge designing made their first appearance in American bridge specifications about fifteen years ago, mostly in the form of the Launhardt and the Launhardt-Weyrauch formulas, or modifications thereof. The previous practice, which is also still largely followed (and sometimes combined with the fatigue formulas) consisted in the variations of the unit stresses for different bridge members, depending upon their position in the bridge, using the lowest unit stresses in the floor system and the highest in the chord members of the trusses. This was done according to the judgment of the designing engineers, out of which gradually had grown certain standards. The exploitation of the fatigue formulas was probably prompted by a desire to bring the arbitrary but well-founded standards, as regards unit stress, under a uniform scientific rule, although that expectation was not fulfilled.

As every one knows, the formulas were deduced by Professors Launhardt and Weyrauch, who were not themselves the experimenters, from certain tests for the Prussian Government, made by Wöhler and Spangenberg about thirty years ago, with steel and iron bars, which were subjected to rapid repetitions and reversions of stress.

The formulas were regarded at the time as examples of elegant mathematical reasoning, and their plausibility won many believers. Nevertheless, their application to bridge designing is absurd, because the conditions on which they are based do not obtain in bridges. This is well recognized by European engineers, who rarely, if ever, use them.

Wöhler strained his iron and steel close up to and over the elastic limit at the rate of 5 to 20 repetitions per second. These conditions may sometimes occur in machinery, car axles, connecting rods, rails, and other like structures, where the tensional and compressional strains are nearly always the effect of sudden and long-continued inter-

mittent forces, and complicated with bending, shearing, or torsional strains. But, even for them, the correctness of the formulas is open to serious question when the stresses are kept far below the elastic limit. Mr. Lindenthal.

Steel rails, made of 80 000-lb. steel, for instance, and subject to fiber stresses from + 27 000 to — 27 000, and higher, from fast and frequent trains, are found to be perfectly safe. They wear out, but they do not break down.

The strains in iron and steel bridges, however, are of an entirely different character. They are primarily static, and in well-designed bridges rarely exceed half the elastic limit of the material. They are not complicated with other strains, as in machinery designing; and, moreover, the maximum strains occur only at long and rare intervals, and in cumulative form.

It surely is not proven, that slow repetitions and reversions of stress, far within the elastic limit of the steel and iron, affect in any way their durability. There is not a single authentic fact or experiment known to support such a contention. Wöhler, who was well qualified to scientifically interpret his own tests, never made such a claim himself.

The values of the Launhardt-Weyrauch formulas agree with Wöhler's tests only partially, within certain limitations of stress. But there are no facts and no tests to prove their applicability to stresses one-half or one-third as low as those which Wöhler used. In the nature of things, such experiments cannot be made. How many thousand years would it take to subject a steel bar of 1 sq. in. to reversions of stress from 12 000 lbs. in tension to 8 000 lbs. in compression, at the rate of one reversion per minute, and at intervals of one hour, before it would suffer in strength, if it ever would? This is assuming, indeed, a higher strain, a quicker reversion and a shorter interval than actually occur in the chords of many existing cantilever and arch bridges, dimensioned according to fatigue formulas. The waste of material which they imply, particularly for members subject to reversion of stress, is not very important for ordinary spans; but for long-span bridges, where the maximum strains occur only under extraordinary and rare conditions, it is a very serious matter, as the bridge is simply loaded down with useless material, adding to its cost, but not to its strength or durability. In the speaker's practice he has never used them, being convinced that the old English practice, also endorsed by Rankine, of separating the strains from dead and live load, and determining the cross-section for both, by using larger unit stresses for the former, and smaller unit stresses for the live load, and then adding the results, is founded on the most rational basis thus far conceived. With the great variations of load conditions and material in steel bridges, it is more than doubtful if the judgment, in that respect, of the experienced designer can ever be replaced in practice by any one or more formulas.

Mr. Ricketts. PALMER C. RICKETTS, M. Am. Soc. C. E.—Some years ago, in discussing the paper "Some Disputed Points in Railway Bridge Designing,"* the speaker predicted that the method of wheel concentrations would be abandoned in a very few years. It is passing into disuse, and that it has not disappeared long ago is to be attributed rather to the conservatism than to the common sense of bridge engineers. It should be abandoned, not because the calculations consume more time and are more difficult than by other methods, but because the method is unscientific, and its accuracy is only apparent. It is irrational and unscientific to obtain stresses by carrying to such extremes calculations which take no account of dynamic effects, while the actual errors in design due to ignorance of dynamic effects are probably many times greater than those due to differences in methods of obtaining stresses. In other words, such calculations will be unnecessary refinements as long as we have to guess at allowed unit stresses.

The subject of fatigue formulas and their use in bridge design was so fully covered in the paper "The Launhardt Formula, and Railroad Bridge Specifications,"† that it is almost unnecessary to discuss it now. About 80% of those who took part in that discussion were of the opinion that fatigue formulas were wholly irrational, and for that reason should not be used.

Mr. Doane. W. A. DOANE, M. Am. Soc. C. E.—The stresses in the various parts of railway bridges can be more easily calculated, and with fully as close an approach to the strains which will actually exist in the structure after completion, by using the method of uniform live loads, rather than by taking certain live loads, which may not obtain with one locomotive out of ten.

The speaker is, therefore, in favor of assuming uniform loads in bridge design, the more so, perhaps, since the general advent of 80-lb. or 90-lb. rails helps materially to distribute the wheel loads on the track stringers, or on the plate floor, as the case may be.

Mr. Morison. GEORGE S. MORISON, Past-President, Am. Soc. C. E.—There is one idea which the speaker has thought over for a good many years. He has always felt that the use of locomotive wheel-base concentration was radically wrong. That it will give results which are not materially bad is certainly true, but the principle of it seems to be very far from what should be followed. It is, briefly, that we take a piece of rolling stock of special weights at special distances and make that a basis for a structure which we expect to last 50 years, and which probably will last 25; and yet that same locomotive will be superseded in 2½ years.

*By J. A. L. Waddell, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. xxvi, p. 77.

†By Henry B. Seaman, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. xli, p. 140.

Mr. Morison.

The original idea has been somewhat improved by using typical locomotives. The specifications of 20 or 25 years ago contained locomotives with the weight on the wheels carried down to the nearest 10 lbs., and the distances between wheels given to the nearest inch—the speaker thinks that some of them went to fractions, but trusts that this was not really the case. The speaker has felt that this was entirely wrong, and a good many years ago he adopted a method which he has always used in drawing his specifications, and which he still thinks gives better results than any system of wheel-base loads.

There is one feature of the wheel-base load which, if drawn out on a diagram, shows its defects. It does not give a uniform line gradually progressing for the varying length of spans, etc., but a line of slight undulations. Now, those undulations ought, in some way, to be eliminated, and something should be obtained which would give a gradual rate of progression from a very short span up to a span of such length that a uniform load can be taken throughout. There are really four different things to be provided in selecting the loads for which railroad bridges are to be proportioned. They are: (1) a heavy wheel-base load; (2) a moderate load for greater lengths; (3) a gradual transition from the heavy wheel-base load down (if measured by weight) or up (if measured by length) to the lighter uniform load; (4) provision for an excess in those members most affected by moving loads.

To these, perhaps, one other should be added. It is simply the provision that for extremely short spans, those less than wheel-base loads, provision should be made for an excessive concentration, and that load ought to be very excessive; it has been the speaker's practice to consider that it was possible that the whole 20 wheel-base loads could be concentrated on two axles, or, perhaps more properly, that half of it could be concentrated on one axle.

It has also seemed expedient to have these provisions governed by coefficients rather than by absolute amounts, and to adapt different structures to loading of different weights, by varying the unit load to which the coefficients are applied.

In other words, there should be a table of coefficients, or a rule for determining the coefficient, for each length of span. Bridges should be classified for the duty they have to perform, and, by applying those coefficients to the unit weight, the weight which is to be used in calculating every length of span could be ascertained at once.

Following this general idea, the speaker adopted, and this was done about 1882, 20 ft. as the wheel-base length and 120 ft. as the length of two locomotives. He then adopted, as the uniform load for 120 ft. and all greater lengths, a weight per lineal foot varying from 2 000 to 4 000 lbs., according to the class of the structure. On the wheel-base length of 20 ft. this unit load was doubled. The transition was effected by adding 1% to the uniform load for each foot of length

Mr. Morison. of structure less than 120 ft., this increase becoming 100% at the wheel-base length of 20 ft. Impact and irregular strains must be provided for chiefly in those members in which variations are rapid and reversals are liable to occur, that is, in the counter members of the web, and to a varying extent in the main members of the web. This extra provision was then made by considering what portion of the strain in any web member would exist under a load equal in intensity to the moving load, but uniform throughout the span. Deducting this from the maximum strain in such member left a remainder which varied from nothing at the end of the span to the full counter strain at the center, and which might be called the variable effect. This variable effect was then estimated on a basis of 1½ times the regular moving load, and gave results which, from their uniform increase from the ends to the center and their general character, seemed to conform with the proper ideas of proportioning a structure.

On this basis, nearly every bridge the speaker has built for the last 16 years has been designed. There are, however, some defects. If applied on the basis of the length of the span, the load for the shearing strains is too low; if applied on the basis of half the length of the

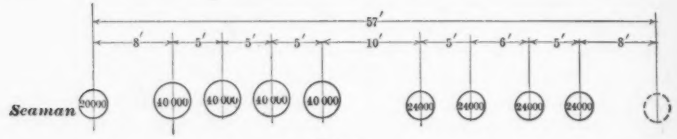


FIG. 1.

span the load for the shearing strains is unnecessarily high. It also gives excessive results for lengths of span midway between 20 and 120 ft.; that is, the transition is made on a straight line from 20 to 120 ft., and it should be made on a curve. This objection has led to the study of a modification of this general scheme which will be shown in the following pages.

The calculations as given are applied to only one unit weight of train, and this has been taken at 5 000 lbs. per lineal foot, which is believed to be the highest unit weight now in use. The result, however, as obtained, is a series of coefficients which can be applied to any unit weight, which unit weight would probably, under present practice, be made to vary from 3 000 to 5 000 lbs.

Take as an illustration of a standard typical locomotive Fig. 1, used in a recent paper in the *Transactions* of this Society.* This engine measures 57 ft. between corresponding points, when coupled up in a train of locomotives. The total weight is 276 000 lbs. This weight has been materially exceeded by recent engines, the two heaviest reported being a consolidation pusher on the Lehigh Valley Railroad,

Transactions, Am. Soc. C. E., Vol. xli, p. 140.

and a consolidation engine running on the Union Railway of Pittsburg, full descriptions of both of which have been given in several of the technical papers. The length is somewhat less than that of the modern consolidation engine, which is usually somewhat over 60 ft., the total length of the Lehigh Valley engine being 62 ft., and that of the Union Railway engine exceeding 65 ft. Mr. Morison.

For purposes of simplicity it would seem advisable to adopt an engine 60 ft. long and to avoid the odd weight on the truck wheels and irregular distances between wheel bearings. These conditions are met by the engine shown in Fig. 2.

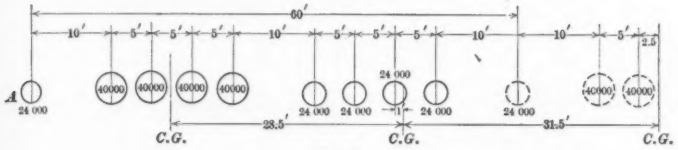


FIG. 2.

It will be observed that the only changes from the engine shown in Fig. 1 are the addition of 4 000 lbs. to the truck wheel, an addition of 2 ft. to the distance between the truck axle and the forward driving axle and the reduction of 1 ft. between the two central tender axles. This engine is 60 ft. long. It weighs 280 000 lbs., 4 000 lbs. more than the other. Every one of the weights is divisible by 4 000 and every distance is a multiple of 5; and yet the strains from this engine would differ so little from those of the engine shown in Fig. 1 that they would not be noticed in any structure that would be built.

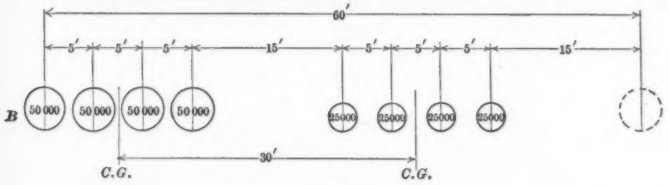


FIG. 3.

The distance between the center of gravity of the driving wheels and the center of gravity of all other wheels is 28.5 ft. A change in this dimension of 1.5 ft. would produce an engine in which the distance from the center of gravity of the four pairs of driving wheels to the center of gravity of all other wheels is half the length of the engine. It will be observed that the truck wheels and tender wheels are taken together in determining the center of gravity. As the weight carried on the truck is largely a matter of adjustment, it seems wise to eliminate the truck wheels in a typical engine and to add their weight to

Mr. Morison. the driving wheels. If these changes are made and the total weight of the engine is increased to 300 000 lbs., or 5 000 lbs. per foot, the result is the typical engine shown in Fig. 3. This engine does not differ materially from that shown in Fig. 4 in any of its effects on a bridge.

Fig. 3 is an exceedingly simple engine so far as its weight on the tracks is concerned. On each of the four driving axles is a weight of 50 000 lbs. On each of the four tender axles is a weight of 25 000 lbs., or just half that of the driving axles; the center of gravity of one set of wheels is just 30 ft. from the center of gravity of the other set of wheels, and every distance is a multiple of 5.

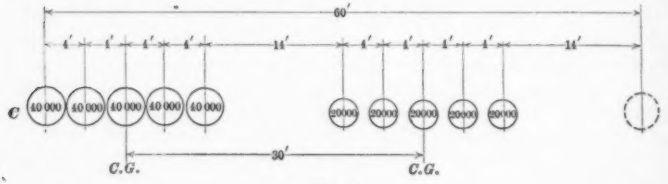


FIG. 4.

Now, this engine does not differ materially, in any of its effects on a bridge, from the engine shown in Fig. 4, which is a ten-wheel coupled locomotive with a tender also having ten wheels. Fig. 4 is a peculiar engine, which has been somewhat used, but it is put in here simply because, though the nominal wheel base is a little more, the actual wheel base is practically the same, and in the effects of the two engines on a bridge hardly any difference can be found.

These engines do not represent any now running, but are typical engines for use in calculation, and producing more excessive effects

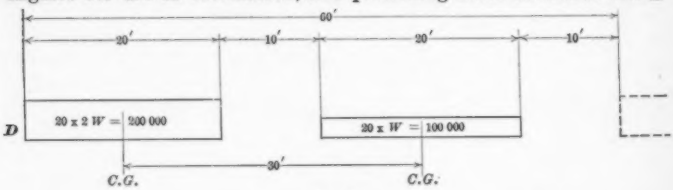


FIG. 5.

than engines of the same weight built on any of the standard lines. Furthermore, they are engines of such form that they can be used very easily in all calculations.

It will be observed that the weight on the driving wheels is exactly double that on the other wheels, that the two sets of wheels are arranged in precisely the same way around their centers of gravity, and that the distance between the two centers of gravity, is half the length of the locomotive, or 30 ft.

If we substitute for the specially applied wheel load, a uniform

Mr. Morison.

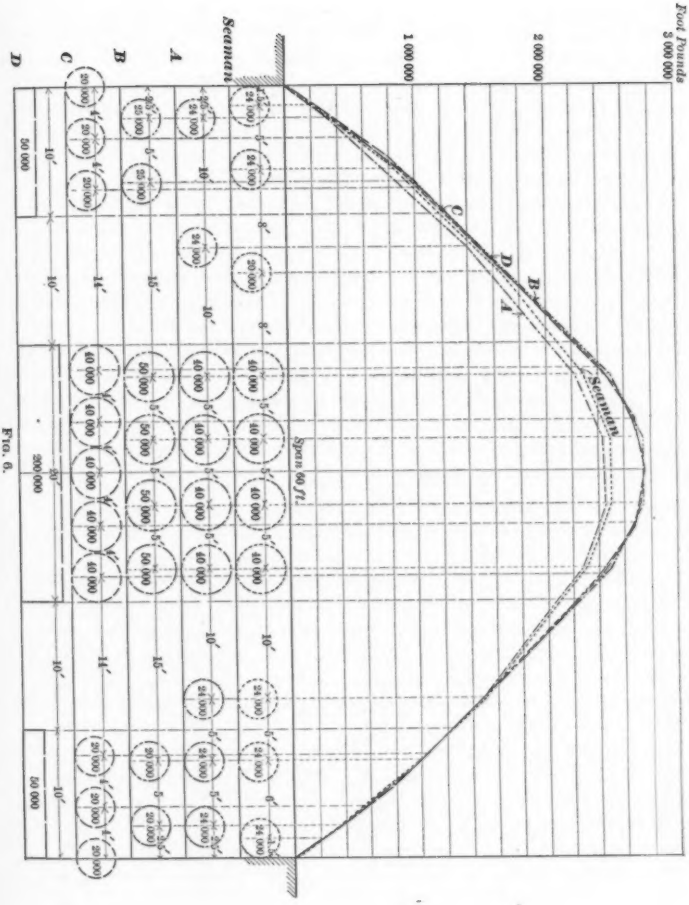


FIG. 6.

Mr. Morison. load of equal intensity, of a length equal to the number of wheels multiplied by the distance between their centers, we have the results shown in Fig. 5.

This loading will also represent passenger locomotives of rather extraordinary dimensions. For instance, the engine wheel-base load of 200 000 may be considered as representing a locomotive with four driving wheels and 50 000 lbs. on each driving wheel; while it is to be hoped that no such locomotive will ever be built, pressures of approximately 50 000 lbs. per wheel now exist in badly counterbalanced engines when running at excessive speeds. That is a thing which bridge specifications have never taken into consideration, and which throws all the ordinary refinements of wheel-base loads out of gear. These strains, however, do not exist on all four wheels at the same time; their influence is felt on single stringers of short lengths, but not on complete spans.

In order to show how closely this typical loading agrees with the several locomotives, Fig. 6 has been prepared, and shows the bending moments under the several conditions. This drawing is taken on the basis of a 60-ft. span with the center of the driving-wheel base placed at the center of the span. The moments of the five different engines and loadings which we have been considering are drawn to scale.

Taking a moving load composed of a continuous train of the typical "*D*" loading (Fig. 5) as a basis of calculation, Table No. 1 shows the maximum shearing strains at the end of the several spans named under such moving load, the lengths of span varying by 20 ft.

TABLE No. 1.

Length of span. Feet.	*a* End shear by typical load.	*b* End shear by uniform load (5 000 lbs. per linear foot).	$a \div b$ Coefficients.
20	100 000	50 000	2.000
40	156 250	100 000	1.563
60	200 000	150 000	1.333
80	250 000	200 000	1.250
100	302 500	250 000	1.210
120	350 000	300 000	1.167
140	400 000	350 000	1.143
160	451 563	400 000	1.129
180	500 000	450 000	1.111
200	550 000	500 000	1.100
220	601 136	550 000	1.093
240	650 000	600 000	1.083
260	700 000	650 000	1.077
280	750 893	700 000	1.073
300	800 000	750 000	1.067
320	850 000	800 000	1.063
340	900 735	850 000	1.059
360	950 000	900 000	1.056
380	1 000 000	950 000	1.053
400	1 050 625	1 000 000	1.051
420	1 100 000	1 050 000	1.048
440	1 150 000	1 100 000	1.045
460	1 200 543	1 150 000	1.044
480	1 250 000	1 200 000	1.042
500	1 300 000	1 250 000	1.040

In the same way the bending moments in the center of the span are given in Table No. 2. Mr. Morison.

TABLE No. 2.

Length of Span. Feet.	a Moment by typical load.	b Moment by uniform load (5 000 lbs. per linear foot).	$a \div b$ Coefficients.
20	500 000	250 000	2.000
40	1 500 000	1 000 000	1.500
60	2 750 000	2 250 000	1.222
80	4 500 000	4 000 000	1.125
100	6 500 000	6 250 000	1.040
120	9 000 000	9 000 000	1.000
140	12 500 000	12 250 000	1.025
160	16 500 000	16 000 000	1.031
180	20 750 000	20 250 000	1.025
200	25 500 000	25 000 000	1.020
220	30 500 000	30 250 000	1.008
240	36 000 000	36 000 000	1.000
260	42 500 000	42 250 000	1.006
280	49 500 000	49 000 000	1.010
300	56 750 000	56 250 000	1.009
320	64 500 000	64 000 000	1.008
340	72 500 000	72 250 000	1.003
360	81 000 000	81 000 000	1.000
380	90 500 000	90 250 000	1.003
400	100 500 000	100 000 000	1.005
420	110 750 000	110 250 000	1.0045
440	121 500 000	121 000 000	1.004
460	132 500 000	132 250 000	1.002
480	144 000 000	144 000 000	1.000
500	156 500 000	156 250 000	1.002

These calculations are based on a train consisting entirely of locomotives, and such trains are not run. Approximations to such trains are found, however, in mineral traffic, in cars of special character. A car designed and built by the Goodwin Patent Gravity Dumping Car Company weighs 42 000 lbs., and is designed to carry 125 000 lbs. of ore, making a total weight of 167 000 lbs. on a length a trifle less than 36 ft., or very nearly 5 000 lbs. per foot. In mineral districts it would seem expedient to proportion bridges for weights equivalent to trains composed entirely of locomotives. In other districts it may be expedient to provide for only two locomotives and to reduce the weight of the following train to something less than the locomotive load. If this reduction is arbitrarily taken at 25%, the train load per foot will be three-quarters of the locomotive load per foot. It seems right to apply this correction in the form of a reduction from a uniform load rather than to make the locomotive an addition to it. On this basis Tables Nos. 3 and 4 have been calculated, Table No. 3 showing the maximum shearing strain at the ends and Table No. 4 the maximum bending moments at the centers of spans, caused by a train consisting of two locomotives followed by a uniform load of three-quarters of the average weight of the locomotives. It will be observed that these two tables are identical with the foregoing tables for all lengths under 120 ft.

Mr. Morison.

TABLE No. 3.

Length of span. Feet.	*a* End shear by typical load.	*b* End shear by uniform load (5 000 lbs. per lineal foot).	$a \div b$ Coefficients.
20	100 000	50 000	2.000
40	156 250	100 000	1.563
60	200 000	150 000	1.333
80	250 000	200 000	1.250
100	302 500	250 000	1.210
120	350 390	300 000	1.168
140	394 075	350 000	1.126
160	436 230	400 000	1.090
180	477 345	450 000	1.061
200	517 734	500 000	1.035
220	557 590	550 000	1.014
240	596 860	600 000	0.994
260	636 238	650 000	0.979
280	675 165	700 000	0.964
300	713 905	750 000	0.952
320	752 500	800 000	0.941
340	791 000	850 000	0.931
360	829 400	900 000	0.921
380	867 555	950 000	0.913
400	905 740	1 000 000	0.906
420	943 860	1 050 000	0.899
440	981 925	1 100 000	0.893
460	1 019 940	1 150 000	0.889
480	1 057 910	1 200 000	0.882
500	1 095 840	1 250 000	0.877

TABLE No. 4.

Length of span. Feet.	*a* Moment by typical load.	*b* Moment by uniform load (5 000 lbs. per lineal foot).	$a \div b$ Coefficients.
20	500 000	250 000	2.000
40	1 500 000	1 000 000	1.500
60	2 750 000	2 250 000	1.222
80	4 500 000	4 000 000	1.125
100	6 523 437	6 250 000	1.044
120	9 037 500	9 000 000	1.004
140	12 260 187	12 250 000	1.001
160	15 718 750	16 000 000	0.982
180	19 413 437	20 250 000	0.959
200	23 336 250	25 000 000	0.933
220	27 490 234	30 250 000	0.909
240	31 875 000	36 000 000	0.885
260	36 240 230	42 250 000	0.858
280	41 583 435	49 000 000	0.849
300	46 837 500	56 250 000	0.833
320	52 500 000	64 000 000	0.820
340	58 687 500	72 250 000	0.812
360	65 250 000	81 000 000	0.805
380	72 187 500	90 250 000	0.800
400	79 500 000	100 000 000	0.795
420	87 185 500	110 250 000	0.791
440	95 250 000	121 000 000	0.787
460	103 687 500	132 250 000	0.784
480	112 500 000	144 000 000	0.781
500	121 687 500	156 250 000	0.779

The coefficients of the four tables have been plotted in Fig. 7.

Mr. Morison.

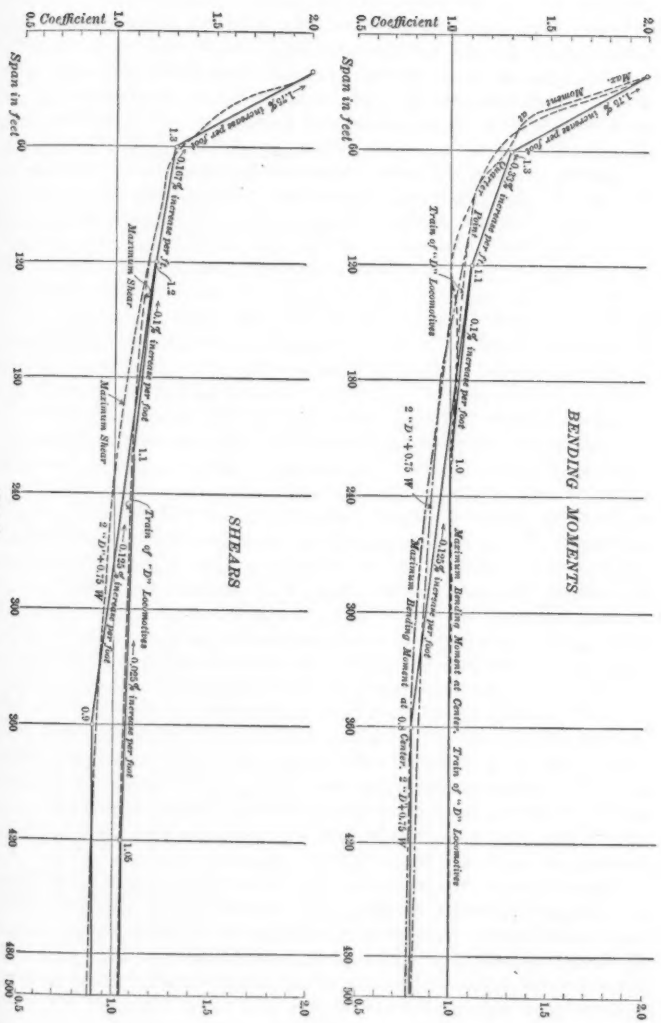


Fig. 7.

Mr. Morison. If these coefficients have to be calculated for every position of loading, there is no advantage in their use; furthermore, the coefficients of Table No. 2 show a series of maxima and minima which ought to be eliminated. To obviate these difficulties a few simple rules are proposed for fixing coefficients for every length of span; the coefficients, as determined by these rules will, in almost all cases, be slightly in excess of the calculated coefficients. The coefficients, as determined by these rules, are plotted as full lines in Fig. 7.

Rule 1.—Bending Moments. Uniform Locomotive Load.—Coefficient for 220 ft., and all greater spans, 1. Increase this coefficient by one-tenth of 1% (0.001), for each foot of decrease of length down to 120 ft., making the coefficient for 120 ft., 1.10. Increase this by one-third of 1% (0.0033), for each foot down to 60 ft., making the coefficient for 60 ft., 1.3. Increase by 1.75% (0.0175), for each foot down to 20 ft., making the coefficient for 20 ft., 2.

Rule 2.—Bending Moments. Two Locomotives Followed by Train.—Coefficient for 360 ft. and all greater spans, 0.8. Increase this coefficient by one-eighth of 1% (0.00125), for each foot of decrease of length down to 120 ft., making the coefficient for 120 ft., 1.1. Increase by one-third of 1% (0.0033), for each foot down to 60 ft., making the coefficient for 60 ft., 1.3. Increase by 1.75% (0.0175), for each foot down to 20 ft., making the coefficient for 20 ft., 2.

Rule 3.—Shearing Strains. Uniform Locomotive Load.—Coefficient for 420 ft. and all greater spans, 1.05. Increase this coefficient by one-fortieth of 1% (0.00025), for each foot of decrease of length down to 220 ft., making the coefficient for 220 ft., 1.1. Increase by one-tenth of 1% (0.001), for each foot down to 120 ft., making the coefficient for 120 ft., 1.2. Increase by one-sixth of 1% (0.00167), for each foot down to 60 ft., making the coefficient for 60 ft., 1.3. Increase by 1.75% (0.0175), for each foot down to 20 ft., making the coefficient for 20 ft., 2.

Rule 4.—Shearing Strains. Two Locomotives in Train Load.—Coefficient for 360 ft. and all greater spans, 0.9. Increase this coefficient by one-eighth of 1% (0.00125), for each foot of decrease of length down to 120 ft., making the coefficient for 120 ft., 1.2. Increase by one-sixth of 1% (0.00167), for each foot down to 60 ft., making the coefficient for 60 ft., 1.3. Increase by 1.75% (0.0175), for each foot down to 20 ft., making the coefficient for 20 ft., 2.

These rules are designed principally as a means of carrying easily and in compact shape these coefficients so that they can be handled quickly, without tabulation. A better method, however, would be to draw a curve governed by the different points which are used in the calculations, and then determine the coefficient for each foot of length from that curve. The speaker has endeavored to find some curve, with a simple formula, which might be used in this way, but, so far, he has not succeeded. Perhaps some other member of the Society may be more fortunate.

Mr. Cowles.

W. L. Cowles, M. Am. Soc. C. E.—The admirable paper entitled "General Criterion for Position of Loads causing Maximum Stress in any Member of a Bridge Truss,*" presented by L. M. Hoskins, Esq., affords an interesting mathematical treatment of the problem, and is a valuable contribution to the theoretical literature on this subject.

The speaker desires, however, at the present time, to discuss briefly the necessity for finding this position, rather than the method of accomplishing the object.

Such a discussion naturally merges with the discussion of concentrated *versus* uniform loads, for the main object to be finally attained is the determination of the maximum stresses produced in the members of a truss by the application of a load within a satisfactory degree of accuracy and with the least labor on the part of the computer.

It must be conceded that a bridge is rarely loaded with the exact load for which it is designed, and in fixing upon the load to be used in making the calculations it is the object of the engineer to specify such a load, either uniform or composed of concentrated wheel-loads, as will produce stresses at least equal to, and preferably somewhat greater than will ever be reached in common practice, making due allowance for any probable increase in loading in the future. This being the case, it follows that extreme accuracy in the calculations is not essential, and, given a satisfactory degree thereof, the problem reduces to a determination of the simplest means of arriving at the results sought.

For ease and quickness of computation it is beyond doubt that a uniform load is preferable, and the speaker is in favor of the adoption of such loads, if railroad engineers can be induced to specify them when requesting propositions for bridge work. Notwithstanding, however, the extended discussion of this subject by the members of this Society, and the offers which have been made of tables indicating equivalent loads, a comparison of the bridge specifications of the railroads of the United States proves that, with a very few exceptions, the engineers of these roads still prefer to specify a loading consisting of a series of concentrated loads representing typical locomotives followed by a uniform train load, and, in the computing department of a bridge works, it becomes necessary to take things as they are rather than as they might be.

It becomes, then, an important matter to determine the simplest manner of preparing the stress sheet, and especially within what limits it is necessary to determine the special position of the load which will produce in each member the maximum stress. With this object in view, the speaker has prepared comparative stress sheets for spans of 125, 150, 175, 200 and 300 ft., based upon loading "*E* 40" of Cooper's

* *Proceedings*, Am. Soc. C. E., March, 1899.

Mr. Cowles. Specifications, the stresses being computed by five different methods, and has tabulated the results showing the differences arising from the use of the respective methods (Table No. 5). The basis of the comparison is Method *A*, in which the position of the load is shifted, in accordance with the formulas of the author, so as to produce the maximum stress in each member. Method *B* is based upon the use of a fixed series of panel loads headed by the maximum floor-beam concentration, progressing on the bridge a panel at a time for successive shears, and covering the entire bridge for all chord moments.

The effect of the small panel load in advance of the maximum head load is disregarded. Method *C* is based upon a uniform load which will produce the same end shear as Method *A*. Method *D* is based upon a uniform load which will produce the same moment at the center as Method *A*. Method *E* is based upon a uniform load equal to the following train load with two additional concentrated loads, two panel lengths apart, which will produce the same end shear as Method *A*. The quantities given are in thousands of pounds, and represent shears not multiplied by the secant in the case of web members, and chord stresses for a depth equal to a panel length, that is, not multiplied by the tangent. A uniform panel length of 25 ft. was adopted.

The allowable limit of variance from the strictly correct stresses which may be permitted for the sake of simplifying the computation is two-fold: First, the percentage of deficiency in any member must not exceed a certain amount; second, the percentage of excess in the total weight of the structure must not exceed a certain amount, and that assumption, for an equivalent load which will result in the smallest percentages of variance and at the same time afford a simple means of preparing the stress sheet, is the one which will naturally be selected. An inspection of the table will show clearly the limit of variance involved in each of these methods, but in general it may be stated that Method *B* gives the smallest percentages, taking both considerations into account. That is, while Method *C* avoids deficient stresses in the most important members, it requires a larger excess of total material; while Method *D* results in a reduction of total weight, the deficiencies in stresses are large; and while Method *E* avoids all deficiencies in stresses, the excess of total weight is even greater than for Method *C*. The maximum deficiency in stress resulting from the use of Method *B* is, for a 150-ft. span, nothing; for a 175-ft. span, 0.1 of 1%; for a 200-ft. span, 2.1% in one member only; for a 300-ft. span, 1.8%; while the excess of total weight is, for the same spans, respectively, 0.8, 0.7, 0.4 of 1% and less than 0.1 of 1 per cent.

The permissible limit of variance is a matter which each engineer will fix for himself in accordance with his own sense of propriety, but it seems to the speaker that the percentages mentioned for Method *B*

Mr. Cowles.

are not excessive, when it is considered that the assumed load itself should be greater than the probable actual load by a margin at least as great as the largest of these percentages. It will be remembered also that, in the case of a new specification coming into a computer's hands, the work involved in Method *B* or its equivalent, must first be done in order to determine the uniform load which may be used in Methods *C*, *D* or *E*, and it is to be noted that still another uniform load must be determined for the floor-beam moment and the stress on the hip vertical. It may be stated here that in determining the percentage of excess weight for Methods *C* and *D* it was assumed that the stress on the hip vertical given in the table would not be used, but that this member would be brought up to proper size as just suggested. The calculation for stringers would, of course, be the same for each method.

If the engine excess for Method *E* be determined by taking the difference between the actual engine weight and the weight of a section of the following load equal in length to the length of the engine, the resulting stresses and the excess weight will be even greater than given in the table.

In case a certain specification is likely to be used frequently, and a method of uniform load is preferred, it is naturally advantageous to prepare a series of uniform loads, corresponding to different lengths of spans, for use in computations; and it cannot be contended that this will not effect a saving in time after the series is once prepared, but the amount of this saving is, in the opinion of the speaker, much overestimated in discussing the relative merits of the different methods. An important factor affecting the simplicity of a computation is the orderly and systematic arrangement of the mathematical work, and in this matter each engineer will adopt such a system as to him will seem the quickest and easiest to obtain the desired result. The speaker has been accustomed to using Method *B*, and the system adopted by him, which is doubtless used by others, affords a reasonably quick process and one which compares favorably with the method of uniform loads. He noted recently the time occupied in making the computations for the truss members of a 300-ft. span with horizontal top chord and having 13 panels, giving odd panel lengths. The load used was "Cooper's *E* 40." Having first determined the panel length, which must be done in any case, the panel loads were determined in 17 minutes and the complete shears and moments, ready for multiplying by secant and tangent, in 18 minutes more, or 35 minutes in all.

There does not seem to be much room here for saving in time by the use of another method, even if the equivalent uniform or excess loads have already been determined.

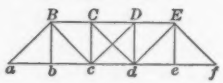
There is nothing new claimed for this method; in fact it may be

Mr. Cowles. called old-fashioned, but this comparison indicates that it gives reasonably close results and is not laborious or slow.

The speaker has not had time to make similar comparisons for subdivided or curved top trusses, but has no reason to suspect that the limits of variance would be any greater in these cases than in those investigated, and he therefore feels confident that, while railroad engineers retain a preference for a type of loading which they can compare readily with actual engines in use on their lines, satisfactory stress sheets may be prepared without undue expenditure of time and labor. He will, however, welcome such a change of attitude, on the part of these engineers, as will result in a reduction in the number of varying loadings and admit of the adoption of a limited number of standard loads, preferably uniform.

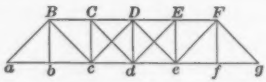
TABLE No. 5.

125-FT. SPAN.



Member.	Method A.	Method B.	Diff.	Per cent.	Method C.	Diff.	Per cent.	Method D.	Diff.	Per cent.	Method E.	Diff.	Per cent.
B C....	194.6	190.0	−4.6	2.4	199.2	+4.6	.....	194.6	.....	.....	193.7	−0.9	0.5
a c.....	132.8	132.8	.....	.....	132.8	.....	.....	129.7	−3.1	2.3	132.8	.....	.....
c d.....	190.0	190.0	.....	.....	199.2	+9.2	.....	194.6	+4.6	.....	193.7	+3.7	.....
B b....	75.6	75.6	.....	.....	66.4	−9.2	12.2	64.9	−10.7	14.2	77.3	+1.7	.....
C c.....	36.4	40.8	+4.4	.....	39.8	+3.4	.....	38.9	+2.5	.....	40.9	+4.5	.....
B a.....	132.8	132.8	.....	.....	132.8	.....	.....	129.7	−3.1	2.3	132.8	.....	.....
B c.....	77.3	81.0	+3.3	.....	79.7	+2.4	.....	77.8	+0.5	.....	81.9	+4.6	.....
C d.....	36.4	40.8	+4.4	.....	39.8	+3.4	.....	38.9	+2.5	.....	40.9	+4.5	.....
Excess weight..			.....	+0.5		.....	+1.1		.....	0.0		.....	+1.0

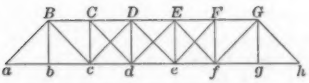
150-FT. SPAN.



Member.	Method A.	Method B.	Diff.	Per cent.	Method C.	Diff.	Per cent.	Method D.	Diff.	Per cent.	Method E.	Diff.	Per cent.
B C....	248.3	248.3	.....	.....	259.2	+10.1	.....	250.8	+2.5	.....	255.5	+7.2	.....
C D....	282.1	282.1	.....	.....	291.6	+ 9.5	.....	282.1	.....	.....	280.5	−1.6	0.6
a c.....	162.0	162.0	.....	.....	162.0	.....	.....	156.7	− 5.3	3.3	162.0	.....	.....
c d.....	248.3	248.3	.....	.....	259.2	+10.1	.....	250.8	+ 2.5	.....	255.5	+7.2	.....
B b.....	75.6	75.6	.....	.....	64.8	−10.8	14.3	62.7	−12.9	17.1	77.8	+2.2	.....
C c.....	62.9	67.5	+4.6	.....	64.8	+ 1.9	.....	62.7	− 0.2	0.3	68.5	+5.6	.....
D d.....	29.7	34.0	+4.3	.....	32.4	+ 2.7	.....	31.4	+ 1.7	.....	34.3	+4.6	.....
B a.....	162.0	162.0	.....	.....	162.0	.....	.....	156.7	− 5.3	3.3	162.0	.....	.....
B c.....	107.6	110.7	+3.1	.....	108.0	+ 0.4	.....	104.5	− 3.1	2.9	111.1	+3.5	.....
C d.....	62.9	67.5	+4.6	.....	64.8	+ 1.9	.....	62.7	− 0.2	0.3	68.5	+5.6	.....
D e.....	29.7	34.0	+4.3	.....	32.4	+ 2.7	.....	31.4	+ 1.7	.....	34.3	+4.6	.....
Excess weight..			.....	+0.8		.....	+1.1		.....	−0.5		.....	+1.2

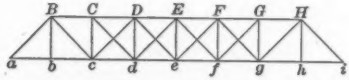
TABLE No. 5—(*Continued*).

175-FT. SPAN.



Member.	Method A.	Method B.	Diff.	Per cent.	Method C.	Diff.	Per cent.	Method D.	Diff.	Per cent.	Method E.	Diff.	Per cent.
B C....	304.8	304.3	−0.5	0.1	316.7	+11.9	.....	305.0	+ 0.2	.....	314.0	+ 9.2	.....
C D....	366.0	366.0	.....	.....	380.0	+14.0	.....	366.0	.....	.....	377.0	+11.0	.....
a c.....	190.0	190.0	.....	.....	190.0	.....	.....	183.0	− 7.0	3.7	190.0	.....	.....
c d.....	304.8	304.3	−0.5	0.1	316.7	+11.9	.....	305.0	+ 0.2	.....	314.0	+ 9.2	.....
d e.....	363.3	366.0	+2.7	.....	380.0	+16.7	.....	366.0	+ 2.7	.....	377.0	+13.7	.....
B b.....	75.6	75.6	.....	.....	63.3	−12.3	16.3	61.0	−14.6	19.3	78.0	+ 2.4	.....
C c.....	90.9	94.9	+4.0	.....	90.5	− 0.4	0.4	87.1	− 3.8	4.2	95.4	+ 4.5	.....
D d.....	52.6	57.9	+5.3	.....	54.3	+ 1.7	.....	52.3	− 0.3	0.6	58.8	+ 6.2	.....
B a.....	190.0	190.0	.....	.....	190.0	.....	.....	183.0	− 7.0	3.7	190.0	.....	.....
B c.....	136.2	138.9	+2.7	.....	135.7	− 0.5	0.4	130.7	− 5.5	4.0	139.1	+ 2.9	.....
C d.....	90.9	94.9	+4.0	.....	90.5	− 0.4	0.4	87.1	− 3.8	4.2	95.4	+ 4.5	.....
D e.....	52.6	57.9	+5.3	.....	54.3	+ 1.7	.....	52.3	− 0.3	0.6	58.8	+ 6.2	.....
E f.....	25.0	29.1	+4.1	.....	27.2	+ 2.2	.....	26.2	+ 1.2	.....	29.4	+ 4.4	.....
Excess weight...	.....	.....	.....	+0.7	.....	.....	+0.9	.....	.....	.....	.....	.....	+1.4

200-FT. SPAN.

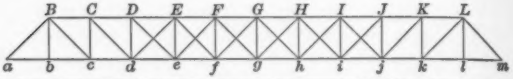


Member.	Method A.	Method B.	Diff.	Per cent.	Method C.	Diff.	Per cent.	Method D.	Diff.	Per cent.	Method E.	Diff.	Per cent.
B C....	359.8	358.7	− 1.1	0.3	372.4	+12.6	.....	355.4	− 4.4	1.2	370.3	+11.1	.....
C D....	448.5	447.6	− 0.9	0.2	465.5	+17.0	.....	444.2	− 4.3	1.0	459.3	+10.9	.....
D E....	473.8	463.8	−10.0	2.1	496.5	+22.7	.....	473.8	.....	.....	484.3	+10.5	.....
a c.....	217.2	217.2	.....	.....	217.2	.....	.....	208.1	− 9.1	4.2	217.2	.....	.....
c d.....	359.8	358.7	− 1.1	0.3	372.4	+12.6	.....	355.4	− 4.4	1.2	370.3	+11.1	.....
d e.....	448.5	447.6	− 0.9	0.2	465.5	+17.0	.....	444.2	− 4.3	1.0	459.3	+10.9	.....
B b.....	75.6	75.6	.....	.....	62.1	−13.7	18.1	59.2	−16.4	21.7	78.1	+ 2.5	.....
C c.....	118.0	121.5	+ 3.5	.....	116.4	− 1.6	1.4	111.0	− 7.0	5.9	121.9	+ 3.9	.....
D d.....	78.4	83.0	+ 4.6	.....	77.6	− 0.8	1.0	74.0	− 4.4	5.6	83.6	+ 5.2	.....
E e.....	45.1	50.6	+ 5.5	.....	46.6	+ 1.5	.....	44.6	− 0.5	1.1	51.5	+ 6.4	.....
B a.....	217.2	217.2	.....	.....	217.2	.....	.....	208.1	− 9.1	4.2	217.2	.....	.....
B c.....	163.9	166.2	+ 2.3	.....	162.9	− 1.0	0.6	156.1	− 7.8	4.8	166.4	+ 2.5	.....
C d.....	118.0	121.5	+ 3.5	.....	116.4	− 1.6	1.4	111.0	− 7.0	5.9	121.9	+ 3.9	.....
D e.....	78.4	83.0	+ 4.6	.....	77.6	− 0.8	1.0	74.0	− 4.4	5.6	83.6	+ 5.2	.....
E f.....	45.1	50.6	+ 5.5	.....	46.6	+ 1.5	.....	44.6	− 0.5	1.1	51.5	+ 6.4	.....
F g.....	21.5	25.5	+ 4.0	.....	23.3	+ 1.8	.....	22.3	+ 0.8	3.7	25.8	+ 4.3	.....
Excess weight..	.....	.....	.....	+ 0.4	.....	.....	+ 1.0	.....	.....	.....	.....	.....	+1.4

Mr. Cowles.

TABLE No. 5—(*Concluded*).

300-FT. SPAN.



Member.	Method A.	Method B.	Diff.	Per cent.	Method C.	Diff.	Per cent.	Method D.	Diff.	Per cent.	Method E.	Diff.	Per cent.
B C....	571.2	569.3	— 1.9	0.3	586.4	+15.2	.....	542.3	—28.9	5.1	585.5	+14.3	.....
C D....	765.1	763.5	— 1.6	0.2	791.6	+26.5	.....	732.1	—33.0	4.3	789.0	+23.9	.....
D E....	899.2	885.0	—14.2	1.6	938.2	+39.0	.....	867.7	—31.5	3.5	933.0	+33.8	.....
E F....	965.3	948.2	—17.1	1.8	1026.2	+60.9	.....	949.0	—16.3	1.7	1017.5	+52.2	.....
F G....	976.2	962.9	—13.3	1.4	1055.5	+79.3	.....	976.2	.....	.....	1042.5	+66.3	.....
a c.....	322.5	322.5	.....	.....	322.5	.....	.....	298.3	—24.2	7.5	322.5	.....	.....
c d.....	571.2	569.3	— 1.9	0.3	586.4	+15.2	.....	542.3	—28.9	5.1	585.5	+14.3	.....
d e.....	765.1	763.5	— 1.6	0.2	791.6	+26.5	.....	732.1	—33.0	4.3	789.0	+23.9	.....
e f.....	899.2	885.0	—14.2	1.6	938.2	+39.0	.....	867.7	—31.5	3.5	933.0	+33.8	.....
f g.....	965.3	948.2	—17.1	1.8	1026.2	+60.9	.....	949.0	—16.3	1.7	1017.5	+52.2	.....
B b.....	75.6	75.6	.....	.....	58.6	—17.0	22.5	54.2	—21.4	28.3	78.5	+ 2.9	.....
C c.....	223.0	225.3	+ 2.3	.....	220.0	— 3.0	1.4	203.4	—19.6	8.8	225.5	+ 2.5	.....
D d.....	179.9	183.0	+ 3.1	.....	175.9	— 4.0	2.2	162.7	—17.2	9.6	183.3	+ 3.4	.....
E e.....	140.9	144.8	+ 3.9	.....	136.8	— 4.1	2.9	126.6	—14.3	10.1	145.2	+ 4.3	.....
F f.....	106.2	110.8	+ 4.6	.....	102.6	— 3.6	3.4	94.9	—11.3	10.6	111.3	+ 5.1	.....
G g.....	76.0	81.0	+ 5.0	.....	73.3	— 2.7	3.6	67.8	— 8.2	10.8	81.5	+ 5.5	.....
B a.....	322.5	322.5	.....	.....	322.5	.....	.....	298.3	—24.2	7.5	322.5	.....	.....
B c.....	270.2	271.8	+ 1.6	.....	268.7	— 1.5	0.6	248.6	—21.6	8.0	271.9	+ 1.7	.....
C d.....	223.0	225.3	+ 2.3	.....	220.0	— 3.0	1.4	203.4	—19.6	8.8	225.5	+ 2.5	.....
D e.....	179.9	183.0	+ 3.1	.....	175.9	— 4.0	2.2	162.7	—17.2	9.6	183.3	+ 3.4	.....
E f.....	140.9	144.8	+ 3.9	.....	136.8	— 4.1	2.9	126.6	—14.3	10.1	145.2	+ 4.3	.....
F g.....	106.2	110.8	+ 4.6	.....	102.6	— 3.6	3.4	94.9	—11.3	10.6	111.3	+ 5.1	.....
G h.....	76.0	81.0	+ 5.0	.....	73.3	— 2.7	3.6	67.8	— 8.2	10.8	81.5	+ 5.5	.....
H i.....	50.4	55.3	+ 4.9	.....	48.9	— 1.5	3.0	45.2	— 5.2	10.3	55.9	+ 5.5	.....
I j.....	29.0	33.7	+ 4.7	.....	29.3	+ 0.3	.....	27.1	— 1.9	6.6	34.5	+ 5.5	.....
Excess weight..	.....	.....	.....	0.0	.....	.....	+ 1.2	.....	.....	.....	.....	.....	+1.7

Mr. Scofield.

E. M. SCOFIELD, Assoc. M. Am. Soc. C. E.—For several years the speaker's time has been devoted to the designing of truss bridges. He has used both methods of calculation, and is not in favor of the method of wheel loads. For each new engine loading it is necessary to make a moment table, which has to be checked carefully, and, up to this point, the two methods are the same. Aside from that, the speaker can compute uniform loads from end shear and figure the stresses in a bridge in about one-quarter to one-tenth of the time required by the exact wheel-load method, thus saving much time and largely avoiding danger from mistakes.

By the uniform method, after checking the end shear, all the stresses in the bridge can be found in from 15 to 20 minutes, and checked beyond all possibility of error. By the wheel-load method the speaker has never been able to make the calculations and check them in such short time; it generally takes from four to ten times as long, and then does not afford such a ready means of checking. In fact, the speaker has seldom made such calculations by wheel loads without making in parallel columns similar calculations by uniform load as a check; and he finds that such a method of checking is more reliable than to attempt to check a wheel-load calculation.

In using the wheel-load method of calculation, the number of manipulations is so great that a man who is figuring with various engines and various specifications is in great danger of making mistakes; whereas the simplicity of the equivalent uniform-load method is such that the calculations can be checked in a very short time. Mr. Scofield.

While there are many different methods of determining equivalent uniform loads, that which bases such loads on the shear in the end panel, is, in the opinion of the speaker, most satisfactory for truss calculations.

All engineers do not agree with this view, but a uniform load based on the shear in the end panel will give stresses which are exact for the end post and nearly exact for the first tie, the error being always on the side of safety and increasing slowly toward the center, and being greatest in the counters where the tonnage involved is small and where an excess of metal is to be commended. This method also gives in the chords a little too much sectional area, but, judging from actual tests of bottom chords, under everyday conditions, a little excess of metal in those members is likely to be well placed.

The argument for exact wheel-loads is altogether too refined, because the engine specified may never pass over the bridge, and the variation due to the use of a uniform load will not exceed 4% in ordinary cases and will not be nearly so much in most of the members where the weight is concentrated.

In the opinion of the speaker, it is well that wheel loads be specified simply because they are more intelligible, especially to the buyer. A railroad engineer who is not dealing with bridges all the time, or the engineer whose time is mostly devoted to other kinds of work, will prefer to have wheel loads specified because they mean something definite to him, which the uniform load does not.

The speaker believes that, in addition to specifying a typical engine or an actual engine, with its following load, a uniform load obtained in a certain, definite way should also be specified and required to be used in making the calculations. The speaker would specify loads based on the center moment for stringers and floor beams and on the moment at the quarter point for girders, and on the shear in the end panel for trusses. This method would give a better bridge and would save time in making the calculations, which would, in the end, be a saving to the buyer. There are other methods for obtaining equivalent loads which are not without merit and are alike in being preferable to the wheel-load method. It would be an additional advantage if the equivalent loads for spans varying by small intervals were given.

HENRY B. SEAMAN, M. Am. Soc. C. E.—Any movement toward the adoption of a train load with a single concentration, instead of what is termed a "typical" engine, but which has really become an equivalent typical engine, should be encouraged. About fifteen years ago it Mr. Seaman.

Mr. Seaman. was the custom to specify all the various types of engines which might in any way produce the maximum strain under varying conditions. Calling to mind especially the Pennsylvania Railroad Specifications, which were presented to the Society about that time by Joseph M. Wilson, M. Am. Soc. C. E., three typical engines were used. These were truly typical, and represented very closely the actual weights and wheel spacing of the various types of engines in use, such as the consolidation, the passenger and the switch engine. But as general specifications were written for adoption upon various roads, it was found expedient to use one engine diagram which would produce strains approximately equivalent to the maximum from the three types. A consolidation engine diagram was outlined for this purpose, and this might properly be termed an equivalent typical engine, *i. e.*, producing strains approximately equivalent to the three types formerly used.

Some ten years ago the speaker had occasion to investigate the weights of engines for C. W. Buchholz, M. Am. Soc. C. E., Chief Engineer of the Erie Railroad, and the result of this study was given in a discussion of the paper on "Some Disputed Points in Railway Bridge Designing.*" There was no engine then in use with a weight on the pony wheel exceeding 16 000 lbs.; and the weights on an axle of the tender in no case would exceed 23 000 lbs., with full weight of water, and with sides boarded up to carry an extra heavy load of coal. To-day the typical engines, or rather the equivalent typical engines, are shown with 20 000 lbs. on the pony wheel, and in some instances with 26 000 lbs. on each tender axle, which is a variation of 10 to 25% from actual conditions. In other words, we are designing bridges for loads which do not exist, and which cannot come upon them.

Mr. Pegram's discussions of the papers "What is the Life of an Iron Railroad Bridge?" and "American Railroad Bridges," as well as reviews in some of the text books, indicate that a uniform train load with a single concentration may produce strains generally within 10% of the engine specified. This would seem about as close as the equivalent typical engine, generally used, would come to the actual load, and, if so, much labor would be saved by the adoption of the single concentration method. The question, therefore, would appear to be, not whether the method of wheel concentration should be continued, but what equivalent load should be used? It should give results approximately equivalent to those obtained in practice; but at the same time the method should be so concise as to permit of its insertion in a specification.

It is, perhaps, unnecessary to discuss further the fatigue formula. The Launhardt formula, which is now used by a number of the larger trunk lines, is a precise interpretation of precise experiments which are never precisely repeated in practice. A study of these same

* *Transactions*, Am. Soc. C. E., Vol. xxvi, p. 77.

experiments indicates that if we consider the destructive effect of live strain as about twice that of dead strain, we will obtain even better general results than by the Launhardt formula, and since this general ratio of 2 to 1 has long been adopted in the better bridge practice, there is no justification for the use of the Launhardt formula. Mr. Seaman.

J. P. SNOW, M. Am. Soc. C. E. (by letter).--The assumptions which it is necessary to make on the matters of typical loads, impact, proper unit strains, fatigue formulas, structural motion, personal predelictions as to column formulas, assessment of riveted members, proper differences for soft and medium steel and many other elements of bridge design which depend on the personal views of the designer, render any hair-splitting nicety absurd, either in computing the strain-sheet or in applying the various rules for deducing sections after the strain-sheet is determined. Mr. Snow.

The railroad official who buys bridges and who does not know how to design them must have a method which he can trust for specifying what he wants, both as to loading and the translation of that load into the final sections of the bridge members.

Without question, a definite engine loading is understood much more readily by such an official than a table or diagram of uniform loads; and Cooper's specification or some other generally accepted one is confidently referred to by him with the supposition that it gives a clear-cut and sufficient set of rules to guide all competing bridge builders in translating the prescribed load into the desired bridge.

The usefulness of the engine loading disappears if the bridge buyer designs his own bridges. The writer has used the uniform-load method exclusively for the last 15 years, both for designing new bridges and examining old ones, and finds it in every way satisfactory and ample, in all cases, to cover the whole ground.

The competing bridge builders who must prepare designs and tenders for the various loadings and specifications offered have the worst of it.

They must find the absolute maximum strains, or the consulting engineer to whom the tenders are referred will throw out their plans; and they must not make such liberal assumptions as will cause them to use more metal than is absolutely needed, else their keener competitors will underbid them. It is a case of full and exact figuring to the last possible permutation with them on every specification offered, and all this with the dreary knowledge that not more than one bid in a dozen will result in an actual contract.

Unless the bridge buyers can be brought to specify uniform loads for their bridges it is difficult to see how the overworked builders can be helped. One set of loads for moments and another for shears, as some have recommended, can hardly appeal to their sense of consistency. A uniform load plus a single rolling load, as many ad-

Mr. Snow. vocate, will not afford them much chance for comparison with familiar structures or train weights.

A single set of uniform loads giving the same center moment as the maximum wheel loads assumed serves as the basis of all the writer's computations and seems to him to be the simplest and most easily comprehended of any that he has seen advocated. The floor-beam load is, of course, the panel length multiplied by the load per foot corresponding to a span equal to twice the panel length. This load is used as a head load in getting the web shears, the following panel loads being those used for the chord moments. By neglecting the partial load ahead of the point under consideration an excessive shear is obtained, but many good designers advocate this practice, and, in the writer's opinion, it is justified, from the fact that under a fast moving train the web members appear to be much more disturbed than the chords.

Shear on longitudinal girders is obtained by using 57% of the total live load for the end reaction and 19% for the center shear. Intermediate shears are proportional to these. The actual maximum shear produced by a set of wheel loads expressed graphically is a flat curve; but the straight line given by the above percentages covers its maximum points and gives a small surplus at the quarter point of the span.

A system of loads for all spans cannot be properly made up from a single class of engines. Heavy passenger engines with two driving axles, or tank shifters on two axles close together, must be used for spans of less than 20 ft., while consolidation engines will govern on long spans. Car loads are so heavy at present that the customary legend of two engines followed by a certain train load may well be abandoned, and a continuous string of typical engines be used in obtaining moments.

It appears to the writer that if prominent bridge engineers could bring themselves to re-write their specifications, giving simple tables or curves of the uniform loads per foot corresponding to various sets of typical wheel concentrations, and give rules for using these loads for all cases, that the bridge buyers would quickly fall into line and the situation would be relieved.

The speaker would vote unreservedly in favor of discontinuing the method of wheel concentrations in determining the stresses in railroad bridges.

Mr Thacher. EDWIN THACHER, M. Am. Soc. C. E. (by letter).—The first specification the writer ever saw giving wheel concentrations as a basis for the calculation of stresses due to live load was issued by the Cincinnati Southern Railroad about 25 years ago. Previous to that time, uniform loads had invariably been used in calculations; unit stresses being varied arbitrarily to suit length of span and location of member.

Mr. Thacher.

This specification gave a live-load diagram of two 10-wheel engines, 36 tons in 12 ft. and 66 tons in 50 ft., followed by 1 820 lbs. per lineal foot. The writer was then employed as Assistant Engineer for the Louisville Bridge and Iron Company, of Louisville, Ky., of which the late Albert Fink, Past-President, Am. Soc. C. E., was President. The specification was received by the morning's mail, together with a letter inviting proposals for several bridges at an early date.

The combined talent of the office was assembled, and that specification was thoroughly discussed. It was voted unanimously that the specification was absurd; that an engine of such weight had never been heard of, and would never be used on the Cincinnati Southern Railroad or any other; that to design bridges for such a load was a reckless waste of money, and that to specify such a load was an outrage. After this feeling had had time to cool off, it was fully realized that this load must be used in calculations, and that the estimates and strain sheets must be prepared speedily, and the all-important question was how to do it. This was before the days of moment tables, or moment diagrams, and before any investigations had appeared giving the exact location of load for the maximum stress in any member.

Late in the afternoon of that day, R. Montfort, M. Am. Soc. C. E., now Chief Engineer of the Louisville and Nashville Railroad, suggested a method of calculation which was at once adopted, and which gave results near enough for all practical purposes. The first driver was placed over a panel point and the panel weights resulting from such location were used in the calculation, and, for double intersection bridges, in which the stresses are somewhat ambiguous at best, not much improvement can be made on that method to-day.

Specifications giving wheel concentrations, once introduced, came rapidly to the front, and in a short time all the railroads were in line, each with a different kind of load, and the bridge companies shouldered their burden, which they have continued to carry ever since.

The writer has been satisfied for many years past that the method of wheel concentrations in the calculation of bridge stresses was a waste of time, and has done what he could to correct the evil. In the specifications of the Keystone Bridge Company of 1887 and 1890 are given tables for equivalent uniform loads for engines of ten different weights, from 44-ton narrow gauge, to 112-ton decapod, and for spans from 10 ft. to 300 ft. in length. For spans of 70 ft. and less (70 ft. being about the maximum length of plate girders used at that time) the uniform loads are such as will give a maximum center moment; and for spans of 75 ft. and upward, such as will give a maximum end shear. These tables have been quite extensively used, and give results practically the same as the wheel loads, and in one-tenth the time. To prepare a moment table for a new loading is some labor, but when

Mr. Thacher.

once prepared, the equivalent uniform load for any span can be found, by aid of the slide rule, in a few minutes.

It was the practice of the writer for some years to calculate bridges, using wheel concentrations, and check by the method of uniform loads, but gradually he came to use uniform loads altogether, except in special cases, generally confined to cases in litigation or reconstruction.

It appears to the writer that the railroad companies would, on proper representation, agree on four or five typical loads, ranging from light to the heaviest in use, some one of which they would be willing to adopt for their bridges. By an expert bridge engineer these could soon be tabulated for convenient use, giving the moments and shears required in girder calculations, and the equivalent loads to be used in the calculation of trusses, thus saving an immense amount of useless labor repeated over and over again. The writer is satisfied that this result will never be accomplished by the efforts of any one engineer, however meritorious his scheme, but believes it can be accomplished by a Special Committee of this Society.

Few engineers believe that Launhardt's formula should be used in designing bridges on account of any fatigue of the material due to repeated application of load, but the writer believes that a modification of this formula is the best thing yet devised for taking account of impact in the distribution of bridge material.

The writer prefers the form, $a\left(1+\frac{\text{Min.}}{\text{Max.}}\right)$. Cooper's specification reduces to $b \div \left(1+\frac{\text{L. L.}}{\text{Max.}}\right)$; and both give practically the same result. It is generally conceded that live load has a worse effect on a bridge than dead load, and the above formulas assume that it has double the effect, but they can be varied to suit any ratio desired, and the results are consistent with each other. The writer could never see any merit in the arbitrary rule-of-thumb methods which add 25% for girders 25 ft. long, and 15% for girders 25 ft. 1 in. long; 50% to this and 10% to that; and he will continue to advocate the above modification of Launhardt's formula until something better is proposed.

Mr. Waddell.

J. A. L. WADDELL, M. Am. Soc. C. E. (by letter).—In 1891 the writer presented to this Society a paper entitled "Some Disputed Points in Railway Bridge Designing,"* in which he advocated the adoption of a few standard train loads for railroad bridges, instead of the numerous ones then in use, offering instead a set of live loads for discussion, and urging that the "Equivalent Uniform-Load Method" be adopted instead of the tedious, burdensome, hair-splitting and heart-breaking method of "Wheel Concentrations."

The thorough discussion which this paper received, and the favor-

* *Transactions*, Am. Soc. C. E., Vol. xxvi, p. 77.

able opinions of most of those who discussed the questions, and of the technical press, induced the writer to carry farther his investigations and to make an earnest endeavor to establish a few standard live loads and the simpler method of computing stresses. Mr. Waddell.

The reviews in the technical press started a series of papers by the writer and others, which were printed in the *Railroad Gazette* and *The Engineering Record*.

These showed conclusively that the "Equivalent Uniform-Load Method" gives results which are accurate enough for all practical purposes, and which are more accurate than those which are obtained by either the "Single-Concentration Method" or the "Double-Concentration Method." Concerning the accuracy of this, there appears to be still some doubt in the minds of a few members of the Society, as shown by opinions expressed in this discussion, so the writer gives herewith certain tabulated results of computations made some years ago for the special purpose of comparing the three alternative methods with the so-called exact method, all tabulated figures having been checked by two computers working entirely independently of each other. The span assumed for comparison was one of 200 ft., which is not far from the average span length for truss bridges in America; and the live loads adopted are those of Classes *Z* and *U* of the writer's "Compromise Standard System of Live Loads for Railroad Bridges."

For trusses, the equivalent uniform load used in each case is an average of those figured for each panel point. This, for nearly all spans, is almost exactly the same as the equivalent for the quarter points.

TABLE No. 6.—PERCENTAGES OF ERROR IN MOMENTS AT CENTERS OF PLATE-GIRDER SPANS, FOUND BY EQUIVALENT-UNIFORM, SINGLE-EXCESS AND DOUBLE-EXCESS SYSTEMS.

Span length. Feet.	Class *Z*.			Class *U*.		
	Equivalent uniform system.	Single-excess system.	Double-excess system.	Equivalent uniform system.	Single-excess system.	Double-excess system.
15....	none.	14.0 safety.	14.0 safety.	none.	19.9 safety.	19.9 safety.
20....	"	7.8 "	7.8 "	"	11.8 "	11.8 "
25....	"	4.2 "	4.2 "	"	5.3 "	5.3 "
30....	"	2.1 "	72.1 "	"	1.2 "	1.2 "
35....	"	0.7 "	0.7 "	"	1.3 danger.	1.3 danger.
40....	"	0.5 "	0.5 "	"	2.3 "	2.3 "
45....	"	0.3 "	0.3 "	"	2.8 "	2.8 "
50....	"	0.0 "	0.0 "	"	3.2 "	3.2 "
60....	"	0.4 danger.	0.4 danger.	"	2.7 "	2.7 "
70....	"	0.1 safety.	0.1 safety.	"	2.2 "	2.2 "
80....	"	0.0 "	0.0 "	"	1.9 "	1.9 "
90....	"	0.1 danger.	0.1 danger.	"	1.7 "	1.7 "
100...	"	0.9 "	0.9 "	"	1.2 "	1.2 "

Mr. Waddell.

TABLE No. 7—(*Continued*).

300.....	Batter brace.....	0.4 danger.	1.4 danger.	0.3 danger.	0.7 safety.	1.5 danger.	1.8 danger.
	1st main diag.....	0.6 "	0.9 "	0.2 safety.	0.1 "	0.8 "	0.8 "
	2d " "	0.6 "	0.0 "	1.1 "	0.7 danger.	0.3 "	0.5 "
	3d " "	0.9 "	0.8 safety.	1.8 "	1.5 "	0.6 safety.	0.1 safety.
	4th " "	1.0 "	2.0 "	2.9 "	2.3 "	1.9 "	1.0 "
	5th " "	0.7 "	4.1 "	4.7 "	2.8 "	4.1 "	2.5 "
	1st counter.....	0.4 safety.	7.5 "	7.5 "	3.0 "	7.4 "	4.7 "
	2d "	1.9 "	12.5 "	11.2 "	2.5 "	12.9 "	8.0 "
	1st top chord.....	0.1 danger.	1.1 danger.	0.1 danger.	0.7 safety.	1.4 danger.	1.4 danger.
	2d " "	0.5 safety.	0.5 "	0.4 safety.	1.0 "	1.2 "	0.9 "
	3d " "	0.7 "	0.3 "	0.3 "	1.6 "	0.6 "	1.0 "
	4th " "	0.6 "	0.4 "	0.3 "	1.8 "	0.4 "	1.3 "
	5th " "	0.1 danger.	1.3 "	0.1 danger.	1.8 "	1.5 "	1.1 "
	Batter brace.....	0.2 "	1.1 "	0.3 "	0.7 "	1.2 "	0.8 "
	1st main diag.....	0.4 "	0.7 "	0.0 safety.	0.4 "	1.0 "	0.7 "
	2d " "	0.5 "	0.3 "	0.4 "	0.2 danger.	0.7 "	0.5 "
	3d " "	0.8 "	0.1 safety.	0.8 "	0.8 "	0.3 safety.	0.2 "
	4th " "	0.9 "	0.6 "	1.2 "	1.4 "	0.3 "	0.3 safety.
	5th " "	1.1 "	1.4 "	1.8 "	2.0 "	1.1 "	0.9 "
	6th " "	1.0 "	2.5 "	2.5 "	2.6 "	2.3 "	1.8 "
	7th " "	0.8 "	4.1 "	5.0 "	3.2 "	3.6 "	2.7 "
	1st counter.....	0.7 "	6.0 "	6.6 "	3.9 "	5.6 "	4.0 "
	2d " "	1.3 safety.	10.5 "	10.5 "	4.9 danger.	9.0 "	6.1 "
400.....	3d " "	0.1 "	1.1 danger.	0.1 danger.	0.7 safety.	1.5 danger.	1.2 danger.
	1st top chord.....	0.3 "	0.9 "	0.1 safety.	0.8 "	1.4 "	1.1 "
	2d " "	0.6 "	0.7 "	0.2 "	0.9 "	1.3 "	1.1 "
	3d " "	0.6 "	0.6 "	0.2 "	1.3 "	1.2 "	1.1 "
	4th " "	0.6 "	0.6 "	0.2 "	1.4 "	0.9 "	0.9 "
	5th " "	0.6 "	0.6 "	0.1 "	1.4 "	0.8 "	0.9 "
	6th " "	0.6 "	0.6 "	0.1 danger.	1.4 "	0.8 "	1.1 "
	7th " "	0.6 "	0.6 "	0.3 "	0.7 "	1.5 "	"
	Bottom chord.....	0.1 danger.	1.3 "				

TABLE No. 7.—PERCENTAGES OF ERROR IN STRESSES, FOUND BY EQUIVALENT-UNIFORM, SINGLE-EXCESS AND DOUBLE-EXCESS METHODS OF LOADING.

Span length.	Chord members.	Web members.	Class *Z*. Equivalent-uniform system.	Class *Z*. Single-excess system.	Class *Z*. Double-excess system.	Class *U*. Equivalent-uniform system.	Class *U*. Single-excess system.	Class *U*. Double-excess system.
100 ft.		Batter brace	0.9 danger.	4.6 safety.	0.9 safety.	1.8 danger.	7.5 safety.	1.2 danger.
		1st main diag.	6.1 safety.	20.2 "	8.5 "	4.5 safety.	26.0 "	5.3 safety.
		1st counter	19.9 "	60.0 "	39.0 "	13.5 "	66.0 "	39.0 "
	1st top chord		3.7 "	9.6 "	0.9 "	4.9	15.0 "	0.8 danger.
	Bottom chord		0.9 danger.	4.6 "	0.9 "	1.8 danger.	7.5 "	1.2 "
150 ft.		Batter brace	1.6 "	0.1 danger.	0.1 danger.	1.1 "	0.9 "	1.7 "
		1st main diag.	0.3 "	4.3 safety.	3.1 safety.	0.6 "	6.0 "	1.3 safety.
		2d "	3.2 safety.	12.6 "	9.0 "	1.4 safety.	15.3 "	6.0 "
		1st counter	10.2 "	30.1 "	17.0 "	7.2 "	35.0 "	12.8 "
	1st top chord		1.1 "	2.8 "	1.7 "	0.6 "	2.5 "	1.4 danger.
	2d "		1.3 "	2.6 "	0.3 "	2.6 "	4.7 "	1.6 "
	Bottom chord		1.6 danger.	0.1 danger.	0.1 danger.	1.1 danger.	0.9 "	1.7 "
200 ft.		Batter brace	1.1 "	1.1 "	0.3 "	0.2 "	0.8 danger.	1.7 "
		1st main diag.	1.0 "	0.7 safety.	1.2 safety.	0.7 "	1.3 safety.	0.3 "
		2d "	0.5 "	3.4 "	3.4 "	2.8 "	2.6 "	0.1 "
		3d "	0.8 safety.	7.8 "	6.7 "	1.3 "	8.9 "	4.1 safety.
		1st counter	5.9 "	18.7 "	14.6 "	2.6 safety.	20.7 "	11.0 "
		2d "	11.8 "	35.5 "	22.2 "	7.5 "	40.3 "	17.0 "
	1st top chord		0.4 "	0.4 "	0.9 "	0.7 "	0.1 "	1.3 danger.
	2d "		0.7 "	0.7 "	0.7 "	1.6 "	1.1 "	1.0 "
	3d "		0.6 "	0.6 "	0.2 danger.	2.2 "	1.7 "	1.4 "
250 ft.		Batter brace	0.8 danger.	1.4 danger.	0.3 "	0.4 "	1.3 danger.	1.5 "
		1st main diag.	0.9 "	0.4 "	0.5 safety.	0.3 danger.	0.3 "	0.7 "
		2d "	0.9 "	0.9 safety.	1.7 "	1.3 "	0.8 safety.	0.1 "
		3d "	0.9 "	2.5 "	3.1 "	2.1 "	2.7 "	1.1 safety.
		4th "	0.3 "	5.5 "	5.5 "	2.4 "	5.9 "	3.1 "
		1st counter	1.9 safety.	11.1 "	9.9 "	1.4 "	11.8 "	6.8 "
		2d "	5.9 "	20.9 "	17.0 "	1.7 safety.	22.9 "	13.0 "
	1st top chord		0.4 "	0.2 danger.	0.8 "	0.6 "	1.1 danger.	1.5 danger.
	2d "		0.5 "	0.1 "	0.6 "	1.1 "	0.6 "	1.3 "
	3d "		0.7 "	0.1 safety.	0.5 "	1.6 "	0.0 "	1.2 "
	4th "		0.5 "	0.1 danger.	0.1 danger.	1.9 "	0.3 safety.	1.5 "

Mr. Waddell. TABLE No. 8.—PERCENTAGES OF ERROR IN END SHEARS, FOUND BY EQUIVALENT-UNIFORM, SINGLE-EXCESS, AND DOUBLE-EXCESS SYSTEMS.

Span length. Feet.	Class *Z*.			Class *U*.		
	Equivalent uniform system.	Single-excess system.	Double-excess system.	Equivalent uniform system.	Single-excess system.	Double-excess system.
15....	none.	1.9 safety.	22.5 danger.	none.	45.4 safety.	3.0 danger.
20....	"	6.0 "	16.7 "	"	33.3 "	10.0 "
25....	"	7.6 "	13.2 "	"	22.8 "	12.3 "
30....	"	5.8 "	12.8 "	"	18.1 "	13.3 "
35....	"	2.8 "	13.9 "	"	14.3 "	14.3 "
40....	"	1.7 "	13.8 "	"	12.0 "	14.4 "
45....	"	0.7 "	13.5 "	"	10.5 "	14.1 "
50....	"	0.6 "	12.5 "	"	9.45 "	13.5 "
60....	"	0.5 "	7·8 "	"	7.9 "	8.4 "
70....	"	1.5 danger.	6.8 "	"	4.0 "	7.6 "
80....	"	3.2 "	6.6 "	"	0.7 "	7.9 "
90....	"	4.3 "	6.4 "	"	1.5 danger.	8.0 "
100...	"	5.1 "	6.4 "	"	2.7 "	7.9 "

In these tables the best possible showing was made for both the "Single-Concentration Method," and the "Double-Concentration Method," as an inspection will show.

The single concentrated load for each class was found by ascertaining, for each main diagonal and each chord section of a 150-ft., a 300-ft., and a 500-ft. span, the single concentrated excess load, which, in addition to the uniform car-load, would produce the same shear or moment as that found by the wheel-concentration method. These excesses for each span were then averaged, and, finally, the average of these averages was taken as the most nearly correct excess load for main-truss members. The results were, respectively, for Classes *Z* and *U*, 39 372 lbs. and 88 368 lbs.

Next, the average single concentrations for all plate-girder spans between 20 ft. and 100 ft. for both classes were determined, and found to be, respectively, 24 577 lbs. and 51 750 lbs. Now, as the floor system is quite as important as the trusses, it was deemed proper to average these concentrations with those found for main-truss members, in order to determine the final single concentration for each class, thus making the said final concentrations, in round numbers, 32 000 lbs. for Class *Z* and 70 000 lbs. for Class *U*.

On applying these to both floor systems and trusses, the errors found were so great as to show conclusively that no single concentration can be assumed which, combined with the constant car load, will give results that even approximate to those found by the concentrated wheel-load method.

It became necessary, therefore, to adopt one concentration for the floor system and another for the trusses, in order to make any show-

ing at all for the system. For Class *Z* there were used 25 000 lbs. for the floor system and 40 000 lbs. for the trusses, and for Class *U* 50 000 lbs. and 90 000 lbs., respectively. On applying these, it was found necessary to make still another compromise, viz., to use the truss concentration in computing end shears in plate-girder spans and in floor systems. Mr. Waddell.

Adopting these compromises, and making calculations thereby, gave the percentages of error shown in Tables Nos. 6, 7 and 8, in the columns headed "Single-Excess system."

The double concentrated loads for the two classes were found by making some extensive calculations. These loads varied from 25 000 lbs. for Class *Z* to 50 000 lbs. for Class *U*. By using these, the percentages of error in Tables Nos. 6, 7 and 8, under the headings "Double-Excess System," were ascertained.

A study of Table No. 7 shows that the equivalent uniform-load method is much more accurate for main-truss members than either the single or the double-concentration method, and, referring to Table No. 6, it is seen that, while the equivalent-uniform load gives exact results (or as nearly so as a due consideration for regularity in the curves will permit), both of the other methods, especially for short spans and stringers of ordinary length, give large errors, notwithstanding the fact that there were adopted different concentrations for floor systems and trusses.

In order to eliminate entirely the personal equation from these tables, Table No. 9 was prepared by collecting the greatest summations of percentages of error for both chords and webs, counters being ignored:

TABLE No. 9.—VARIATIONS IN PERCENTAGES OF ERROR, AS GIVEN BY EQUIVALENT-UNIFORM, SINGLE-EXCESS AND DOUBLE-EXCESS SYSTEMS.

Span Length. Feet.	Members.	Class *Z*.			Class *U*.		
		Equivalent uniform system.	Single-excess system.	Double-excess system.	Equivalent uniform system.	Single-excess system.	Double-excess system.
100..	Chord members.	4.6	5.0	.....	6.7	7.5	0.4
	Web "	7.0	15.6	7.6	6.3	18.5	6.5
150..	Chord "	0.2	0.2	1.4	2.0	2.2	0.2
	Web "	4.8	12.7	9.1	2.5	14.4	7.7
200..	Chord "	0.3	0.3	1.1	1.5	1.6	0.4
	Web "	1.9	8.9	7.0	2.6	9.7	5.8
250..	Chord "	0.3	0.3	0.9	1.3	1.4	0.3
	Web "	0.6	6.9	5.8	2.8	7.2	4.6
300..	Chord "	0.8	0.8	0.5	1.1	1.1	0.5
	Web "	0.6	5.5	5.0	3.5	5.6	3.8
400..	Chord "	0.5	0.5	0.3	0.7	0.7	0.3
	Web "	1.0	3.8	3.8	3.9	3.9	2.9

When the extreme errors were of opposite kinds, their arithmetical sum was taken, but when they were of the same kind their numerical difference was taken.

A study of Table No. 9 shows that, as far as chord stresses are con-

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cerned, there is but little to choose from between the three methods, but that, in respect to web members, the equivalent uniform-load method is very much more accurate than either the single-excess or the double-excess method.

Summarizing, it is evident that the following have been proved:

1st. The equivalent uniform-load method, for all bridges of types sanctioned by the best modern practice, gives stresses agreeing sufficiently well for all practical purposes with those found by the concentrated wheel-load method.

2d. That there is no combination of a constant car load per lineal foot and a single concentrated load which will give, for all members of trusses and floor systems, stresses which will agree even approximately with those found by the concentrated wheel-load method.

3d. That even if for any system of loading two separate single concentrations be adopted, one for floor systems and the other for main-truss members, the said single concentrations cannot be adjusted so as to give results agreeing sufficiently well for practical purposes with those found by the concentrated wheel-load method.

4th. That if two engine excesses placed about 50 ft. apart at panel points be adopted in connection with a constant car load per lineal foot, the stresses obtained thereby in members of floor systems and trusses do not agree sufficiently well, for practical purposes, with those obtained by the concentrated wheel-load method.

Now, on the strength of this showing, what is the use of talking any more about using a constant car load per lineal foot headed by either one or two concentrations? Either of the last two methods requires, in making calculations of stresses, fully double the time and labor required by the equivalent uniform-load method, and is by no means as accurate.

There is no method of computing live-load stresses so simple and easy as that which involves the use of properly figured uniform loads, so, why not adopt it for good and all, if we are to abandon in future the use of the "Wheel-Concentration Method"?

And surely the time has come to abandon the latter method forever, if one may judge by the unanimous opinion of those who took part in the oral discussion of the question.

In respect to Mr. Morison's proposed method, it must be evident to all that instead of simplifying matters it would complicate them worse than ever.

The objections which railroad engineers offer to using live loads which they cannot understand readily, and which, at least in their minds, do not approximate actual train loads, is well founded.

Let each railroad engineer who desires to have a bridge designed choose a combination of two locomotives and a train of cars of indefinite length weighing a certain number of pounds per lineal foot, and, on the same diagram upon which this loading is drawn to scale, let there be

drawn also the curves for equivalent loads and for end shears in plate-girder spans; and let there be written thereon a note to the effect that the curves are to be used instead of the actual wheel concentrations. Mr. Waddell.

But, instead of each engineer having a live load of his own, why not provide a variety of live loads differing from each other by uniform gradations from which anyone can choose a load to suit his requirements?

"In November, 1892, the author sent a circular-letter to all the chief engineers of railroads in the United States and Canada who were members (in any grade) of the American Society of Civil Engineers, and to every other member of that society connected with or specially interested in the designing, building or operating of railroad bridges. This letter solicited a ballot on certain 'Disputed Points in Railway Bridge Designing,' foremost among which were those of standard live loads and a simple equivalent method for computation. The number of responses received was as great as could have been expected; and the result was that about 82% of those who voted favored, and 18% opposed, the adoption of 'A Standard System of Live Loads for Railway Bridges' similar to that proposed by the author. Eighty-two per cent. also of those who voted were in favor of abandoning the 'Concentrated Wheel-Load Method,' and 18% were in favor of retaining it. Of the former, 78% favored the 'Equivalent-Uniform Load Method,' and 22% were in favor of either the 'Single' or the 'Double Concentration Method.' A number of gentlemen who responded made valuable suggestions in respect to the standard system of live loads propounded, and by the aid of these the author prepared a proposed 'Compromise Standard System of Live Loads for Railway Bridges,' and submitted the same, as before, for final ballot in May, 1893.

"The number of replies received showed that great interest was taken in the question; and the result of the ballot was 90% in favor and 10% opposed to the proposed standard.

"Next the pamphlet was published and distributed quite generally among those engineers interested in the subject of bridges, a copy being sent not only to every one who had replied to the ballots, but also to every railroad chief engineer in the United States, Canada and Mexico whose address was given in *Poor's Manual*. To these chief engineers there was also sent another circular-letter with a ballot that read as follows:

"'I $\left\{\begin{matrix}\text{agree}\\\text{do not agree}\end{matrix}\right\}$ to use the "Compromise Standard System of Live Loads for Railway Bridges" when calling for bids on railroad-bridge work, or when having plans prepared for railroad bridges.'

"'I $\left\{\begin{matrix}\text{agree}\\\text{do not agree}\end{matrix}\right\}$ to specify that the "Equivalent Uniform-Load Method" is to be used in computing stresses in the bridges that are to be designed for my road.'

"Over one hundred chief engineers thus addressed voted in favor of both propositions, and very few were opposed."

The Compromise Standard System of Live Loads for Railroad Bridges has now been in use for six years, and all who have used it and its equivalents endorse heartily both the loads themselves and the equivalent method of calculating stresses.

Unfortunately, locomotives and cars have both grown somewhat heavier since the writer's pamphlet was issued, so that it is now neces-

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sary to add to the live loads "Class *S*," for which the axle loads would be as follows:

Pilot axle	22 000 lbs.
Driving axles	43 000 lbs. each.
Tender axles	25 000 lbs. each.
Cars	4 400 lbs. per lineal foot.
Alternative axle load	54 000 lbs. each, on two axles only.

The equivalent curves can be drawn on the diagrams by eye, correctly enough for all practical purposes, by simply extending upward the spacing of the curves.

There is one point in connection with all these equivalent curves to which the writer desires to call attention, viz., the equivalent loads which they represent were figured for cars preceding as well as following the locomotives, in the case of chord stresses.

The writer desires also to call attention to the short method, evolved by him some eight years ago, of computing for any truss, by one simple graphic diagram and by the use of a slide rule, all the main-member stresses, both direct and reverse.

The method is as follows:

Assume that there is an upward reaction at one end of the span equal to 100 000 lbs., caused by an unknown load placed at the first panel point from the other end of the span, then find graphically the stress in each web-member from end to end of span, caused by this assumed upward reaction. Then calculate the value of the live-load reaction for the maximum stress in each web-member by means of the slide rule and the following formula and table in which n is the number of panels in the span, n' is the number of the panel point at the head of the train, counting from the loaded end of the span, and C is the coefficient of $\frac{L}{n}$.

Live-load reaction for the head of train at $n' = C \times \frac{L}{n}$.

n'	C	n'	C	n'	C	n'	C
1	1	7	28	13	91	19	190
2	3	8	36	14	105	20	210
3	6	9	45	15	120	21	231
4	10	10	55	16	136	22	253
5	15	11	66	17	153	23	276
6	21	12	78	18	171	24	300

Then, still using the slide rule, find the greatest live-load stress in each web-member by the following equation:

$$\text{Stress required} = \text{Stress from Assumed Reaction} \times \frac{\text{Actual Reaction}}{\text{Assumed Reaction}}$$

Where the panels are divided, as in the Petit truss, and where

inclined sub-posts are used, the tensile stress in the upper half of each main diagonal thus found will have to be corrected by subtracting therefrom a stress equal to $\frac{L}{2}$ Sec. A, where A is the inclination of the diagonal to the vertical. But when inclined sub-ties are used instead of inclined sub-posts, the correction just referred to will apply only to the compressive stresses in the lower halves of the main diagonals. The reason for making this correction, as will be at once evident to any one who is accustomed to finding stresses in Petit trusses, is that the method above outlined ignores the subdivision of the panels when ascertaining by graphics the stresses caused by the assumed upward reaction. Mr. Waddell.

The writer's sole reason for again bringing forward this quick method is that he would like bridge engineers to learn to use it and thus save themselves much valuable time and energy, as it cuts down the work of computing web stresses to a very small percentage of the amount usually expended thereon.

In conclusion, it appears from the opinions advanced in the oral discussion, that the time has come to drop from engineering practice and to relegate to text books, where it can still serve as an excellent example of mental gymnastics, the pseudo-exact, laborious and tedious method of Wheel Concentrations, and to adopt instead some simpler methods; consequently, the writer herewith proposes that the "Compromise Standard System of Live Loads for Railroad Bridges" and the "Equivalent Uniform-Load Method" of computing stresses be adopted by the profession for general practice.

O. E. Selby, Jun. Am. Soc. C. E. (by letter).—It is the purpose of the writer to present, first, three tables giving actual moments and shears in trusses and plate-girders produced by the loading in "Cooper's 1896 Specifications;" second, a table of the single-excess loads which, with the constant uniform train-load, will produce the same moments and shears as the wheel-loads; third, two diagrams on which these excess loads are plotted in order to determine their laws of variation; fourth, three simple formulas deduced from the diagrams, which will give values of excess loads producing moments and shears substantially equal to those produced by the wheel loads; fifth, a simple specification for loading which will combine in one operation the computing of the quiescent stresses and the allowance for impact. Mr. Selby.

The tables for moments and shears for Cooper's "$E40$" have been computed systematically and checked carefully, and will be useful to those who still prefer to, or find it necessary to, compute stresses by the actual wheel-loads for this specification.

Table No. 10 gives the moment, end shear, center shear and shear at quarter point for plate-girder spans, varying by increments of 1 ft., from 10 to 125 ft. Tables Nos. 11 and 12, respectively, give moments and shears for truss spans of 5, 6, 7, 8 and 9 panels, and panel lengths

Mr. Selby. varying by increments of 1 ft. from 20 to 35 ft. This includes span lengths between 100 ft. and 315 ft. The shears apply only to trusses with parallel chords. The arrangement of the tabulation by panel lengths rather than span lengths is believed to be new, and facilitates a complete tabulation in a small space. Values for intermediate panel lengths can be obtained by direct interpolation. All values in the tables are for one truss or girder.

Having prepared the foregoing tables for his own use, some time ago, the writer began to cast about for a simple substitute loading which could be used instead of wheel loads in general without material variation in the results, and for some rule for varying the substitute loading to correspond with different engine loadings. He made many comparative computations using various single excesses, double excesses, uniform loads, etc., with results in favor of the single-excess method, but without finding any one excess load which gave satisfactory results for widely different span lengths. Finally, it occurred to him to compute, for the various positions and span lengths, the excess loads which, with the uniform train-load, would produce the actual moments and shears in the table, and to study their laws of variation.

Table No. 13 gives the values of such excess loads E, for one truss or girder, for all panel points of trusses of 5, 6, 7, 8 and 9 panels, with panel lengths varying by 3 ft. from 20 to 35 ft., and for plate-girders of lengths varying by 5 ft. from 10 to 125 ft.

The formulas for computing E are very simple. Let M = total actual moment, M_u = moment from uniform load and M_e = moment from excess load. Then $M = M_u + M_e$. Similarly for shears $S = S_u + S_e$.

Let w = uniform train-load per foot and l = span.

For moment in a plate-girder:

$$M = \frac{1}{8} w l^2 + \frac{1}{4} E l, \text{ from which}$$

$$E = \frac{4 M}{l} - \frac{w}{2} l.$$

For a train-load of 4 000 lbs. per foot, with 1 000-lb. units, $w = 2$ for one girder, then $E = \frac{4 M}{l} - l.$

For end shear in a plate-girder:

$$S = \frac{1}{2} w l + E, \text{ from which } E = S - \frac{1}{2} w l.$$

If $w = 2$, $E = S - l$.

For center shear in a plate-girder:

$$S = \frac{1}{4} w \frac{l}{2} + \frac{E}{2} = \frac{1}{8} w l + \frac{E}{2}, \text{ from which}$$

$$E = 2 S - \frac{1}{4} w l.$$

If $w = 2$, $E = 2 S - \frac{1}{2} l$.

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For moments in a truss, let N = whole number of panels and n = number of panels to the left of the point in question; then, considering the uniform panel load concentrated at the panel point,

$$M_u = \frac{N-1}{2} p\, w \times n\, p - \frac{n\,(n-1)}{2} p \times p\, w = p^2 \frac{w}{2} n\,(N-n).$$

If $w = 2$, $M_u = p^2\, n\,(N-n)$.

$$M_e = \frac{N-n}{N} E\, n\, p$$

$$M = p^2\, n\,(N-n) + \frac{N-n}{N} E\, n\, p = n\,(N-n)\, p \left(p + \frac{E}{N}\right), \text{ from}$$

which

$$E = N \left(\frac{M}{n\,(N-n)\, p} - p\right)$$

For shears in a truss:

$$S_u = p\, w \frac{(N-n+1)\,(N-n)}{2N}.$$

If $w = 2$, $S_u = p \frac{(N-n+1)\,(N-n)}{N}$.

$$S_e = \frac{N-n}{N} E.$$

$$S = p \frac{(N-n+1)\,(N-n)}{N} + \frac{N-n}{N} E, \text{ from which}$$

$$E = \frac{N}{N-n} S - p\,(N-n+1).$$

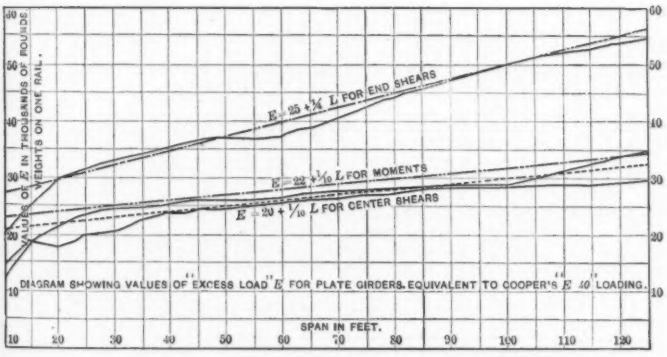
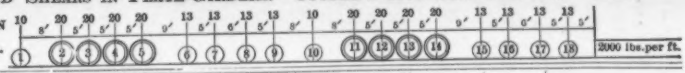

VALUES OF E IN THOUSANDS OF POUNDS PER LINEAL FOOT OF LOAD ON ONE RAIL.
E = 25 + 3/4 L FOR END SHEARS
E = 22 + 1/10 L FOR MOMENTS
E = 20 + 1/10 L FOR CENTER SHEARS
DIAGRAM SHOWING VALUES OF EXCESS LOAD "E" FOR PLATE GIRDERS EQUIVALENT TO COOPER'S E 40 LOADING.
SPAN IN FEET.
10 20 30 40 50 60 70 80 90 100 110 120

FIG. 8.

An inspection of Table No. 13 shows that the values of E increase slowly with the span length for plate-girders and are nearly uniform for different panel lengths, but vary with the position in the truss and the number of panels. The values for plate-girders have been plotted on the diagram, Fig. 8, together with straight lines, which coincide

TABLE No. 10.—MAXIMUM MOMENTS AND SHEARS IN PLATE-GIRDERS. COOPER'S 1896 SPECIFICATIONS "E 40."

MOMENTS IN THOUSANDS OF FT.-LBS. SHEARS IN THOUSANDS OF LBS. WEIGHTS ON ONE RAIL.

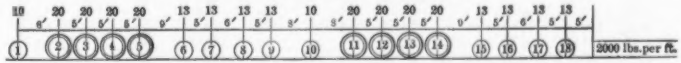


Span.	Moment.	Shears. End.	Shears. ¼ Pt.	Shears. Center.	Span.	Moment.	Shears. End.	Shears. ¼ Pt.	Shears. Center.	Span.	Moment.	Shears. End.	Shears. ¼ Pt.	Shears. Center.	Span.	Moment.	Shears. End.	Shears. ¼ Pt.	Shears. Center.
.....	.....	.....	.....	.....	36.....	548.6	70.6	44.1	20.6	66.....	1 539.5	105.0	63.6	29.7	96.....	2 993.9	144.8	83.3	38.3
.....	.....	.....	.....	.....	37.....	574.3	71.9	44.8	21.0	67.....	1 581.5	106.4	64.2	30.0	97.....	3 049.0	146.2	84.2	38.5
8.....	37.8	27.5	17.5	10.0	38.....	600 0	73.1	45.4	21.3	68.....	1 623.5	107.8	64.8	30.2	98.....	3 106.5	147.5	85.0	38.8
9.....	46.9	28.9	18.9	10.0	39.....	626.6	74.3	46.0	21.7	69.....	1 665.5	109.2	65.4	30.5	99.....	3 162.3	148.8	85.8	39.1
10.....	56.3	30.0	20.0	10.0	40.....	655.6	75.4	46.8	22.0	70.....	1 706.9	110.5	65.9	30.7	100.....	3 219.9	150.0	86.6	39.4
11.....	65.7	32.7	20.9	10.9	41.....	684.6	76.8	47.5	22.3	71.....	1 749.3	111.8	66.5	31.1	101.....	3 277.6	151.2	87.3	39.6
12.....	80.0	35.0	21.7	11.7	42.....	713.6	78.1	48.2	22.6	72.....	1 792.2	113.3	67.0	31.4	102.....	3 335.9	152.4	88.1	39.9
13.....	95.0	36.9	22.3	12.3	43.....	742.6	79.4	48.9	22.9	73.....	1 833.9	114.8	67.5	31.7	103.....	3 410.6	153.7	88.8	40.1
14.....	110.0	38.6	23.6	12.9	44.....	771.6	80.6	49.5	23.2	74.....	1 879.2	116.3	68.0	32.0	104.....	3 475.2	154.9	89.5	40.4
15.....	125.0	40.0	25.0	13.3	45.....	800.6	81.7	50.1	23.4	75.....	1 925.8	117.7	68.6	32.3	105.....	3 537.6	156.1	90.3	40.6
16.....	140.0	42.5	26.3	13.8	46.....	829.6	82.8	50.7	23.7	76.....	1 972.0	119.1	69.2	32.6	106.....	3 600.3	157.3	90.9	40.9
17.....	155.0	44.7	27.4	13.8	47.....	858.6	83.8	51.4	23.9	77.....	2 019.1	120.4	69.9	32.9	107.....	3 666.6	158.5	91.7	41.1
18.....	170.0	46.7	28.3	13.9	48.....	887.6	84.8	52.1	24.2	78.....	2 064.4	121.7	70.5	33.2	108.....	3 745.3	159.6	92.4	41.3
19.....	186.6	48.4	29.2	14.0	49.....	918.8	86.0	52.8	24.5	79.....	2 112.3	123.0	71.1	33.4	109.....	3 818.4	160.8	93.2	41.6
20.....	206.3	50.0	30.0	14.0	50.....	950.9	87.1	53.5	24.9	80.....	2 160.5	124.2	71.7	33.7	110.....	3 886.8	162.0	93.9	41.8
21.....	226.0	51.4	31.4	14.5	51.....	983.1	88.2	54.1	25.2	81.....	2 207.7	125.6	72.3	34.0	111.....	3 958.2	163.2	94.6	42.0
22.....	245.7	52.7	32.7	15.0	52.....	1 015.2	89.2	54.8	25.5	82.....	2 256.7	126.9	73.0	34.4	112.....	4 026.9	164.4	95.3	42.2
23.....	265.4	53.9	33.9	15.4	53.....	1 047.4	90.2	55.4	25.8	83.....	2 306.5	128.2	73.7	34.7	113.....	4 099.0	165.5	96.0	42.5
24.....	285.2	55.0	35.0	15.8	54.....	1 081.4	91.2	56.1	26.1	84.....	2 356.3	129.5	74.4	35.0	114.....	4 172.0	166.7	96.8	42.8
25.....	305.0	56.5	36.0	16.2	55.....	1 116.9	92.1	56.8	26.4	85.....	2 406.9	130.7	75.1	35.3	115.....	4 245.0	167.9	97.5	43.1
26.....	324.8	57.9	36.9	16.5	56.....	1 152.4	93.0	57.5	26.6	86.....	2 459.6	132.1	75.8	35.6	116.....	4 318.8	169.0	98.3	43.4
27.....	344.6	59.2	37.8	16.9	57.....	1 187.9	94.2	58.2	26.9	87.....	2 510.6	133.4	76.5	35.9	117.....	4 389.5	170.2	99.0	43.7
28.....	365.5	60.4	38.6	17.1	58.....	1 223.4	95.4	58.8	27.2	88.....	2 564.2	134.7	77.1	36.2	118.....	4 463.8	171.4	99.7	43.9
29.....	388.0	61.6	39.3	17.4	59.....	1 261.0	96.5	59.5	27.5	89.....	2 615.9	136.0	77.9	36.5	119.....	4 538.8	172.5	100.4	44.2
30.....	410.5	63.0	40.0	17.7	60.....	1 299.6	97.6	60.1	27.9	90.....	2 670.5	137.2	78.7	36.7	120.....	4 614.1	173.7	101.1	44.5
31.....	432.9	64.4	40.7	18.2	61.....	1 338.3	98.7	60.7	28.2	91.....	2 723.0	138.5	79.5	37.0	121.....	4 686.5	174.8	101.8	44.7
32.....	455.4	65.7	41.3	18.8	62.....	1 377.0	100.0	61.3	28.5	92.....	2 775.9	139.8	80.3	37.3	122.....	4 762.7	176.0	102.5	45.0
33.....	477.9	66.9	42.0	19.2	63.....	1 415.6	101.3	61.8	28.8	93.....	2 831.5	141.1	81.0	37.5	123.....	4 836.2	177.1	103.2	45.3
34.....	500.4	68.1	42.8	19.7	64.....	1 455.5	102.6	62.4	29.1	94.....	2 885.3	142.4	81.7	37.8	124.....	4 917.4	178.3	104.0	45.7
35.....	523.0	69.2	43.5	20.1	65.....	1 497.5	103.8	63.0	29.4	95.....	2 939.5	143.6	82.5	38.0	125.....	4 996.4	179.4	104.7	46.0

Mr. Selby.

Mr. Selby.

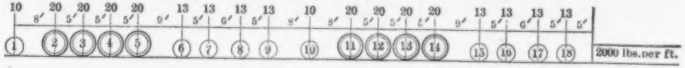
TABLE No. 11.—MAXIMUM MOMENTS. COOPER'S 1896 SPECIFICATION "E 40." MOMENTS IN THOUSANDS OF FOOT-POUNDS. WEIGHTS ON ONE RAIL.



Panel Points.	Panel Lengths.																
	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	
1/5	2 186.2	2 400.4	2 622.4	2 849.8	3 081.8	3 320.2	3 565.0	3 816.2	4 073.8	4 337.8	4 608.2	4 890.0	5 181.2	5 478.8	5 782.8	6 093.2	
1/6	2 689.3	2 942.7	3 206.8	3 479.3	3 760.2	4 049.3	4 346.8	4 652.7	4 966.8	5 289.3	5 620.2	5 959.3	6 312.7	6 676.8	7 049.3	7 430.2	
1/7	3 163.7	3 453.4	3 761.3	4 080.1	4 409.3	4 748.7	5 098.4	5 458.4	5 828.7	6 209.3	6 600.1	7 001.3	7 413.4	7 843.7	8 284.3	8 735.1	
1/8	3 619.5	3 951.1	4 298.1	4 663.0	5 040.1	5 429.5	5 831.1	6 245.0	6 671.1	7 109.5	7 560.1	8 023.0	8 498.1	8 991.1	9 499.5	10 020.1	
1/9	4 062.9	4 436.2	4 823.8	5 233.9	5 658.8	6 097.9	6 551.2	7 018.8	7 500.6	7 996.6	8 506.8	9 031.2	9 569.9	10 125.6	10 701.6	11 291.8	
2/5	3 154.4	3 477.6	3 813.6	4 154.4	4 512.0	4 874.4	5 241.6	5 613.6	5 993.4	6 387.0	6 785.4	7 188.6	7 596.6	8 009.4	8 472.4	8 953.6	
2/6	4 204.7	4 599.3	4 999.3	5 404.7	5 815.3	6 234.7	6 682.3	7 167.7	7 663.7	8 173.7	8 704.3	9 245.7	9 797.7	10 381.7	10 981.7	11 608.3	
2/7	5 060.6	5 531.4	6 024.3	6 537.9	7 073.6	7 623.6	8 187.9	8 783.6	9 393.6	10 017.9	10 664.3	11 331.4	12 012.9	12 715.4	13 448.3	14 204.6	
2/8	5 966.3	6 530.3	7 112.3	7 712.3	8 344.0	8 995.0	9 664.0	10 360.0	11 083.0	11 824.0	12 584.3	13 376.3	14 186.3	15 014.3	15 880.3	16 768.3	
2/9	6 862.4	7 503.1	8 173.6	8 865.8	9 588.1	10 339.4	11 112.6	11 910.1	12 745.4	13 602.6	14 481.4	15 393.8	16 331.8	17 291.6	18 285.6	19 313.8	
3/6	4 596.5	5 057.0	5 536.5	6 030.0	6 532.5	7 056.5	7 592.0	8 136.5	8 690.0	9 252.5	9 836.5	10 432.0	11 036.5	11 650.0	12 272.5	12 916.5	
3/7	6 117.1	6 677.7	7 262.9	7 861.1	8 487.3	9 155.9	9 850.3	10 563.4	11 290.3	12 030.9	12 787.3	13 572.4	14 371.3	15 183.9	16 010.1	16 859.4	
3/8	7 387.6	8 056.5	8 795.4	9 581.0	10 385.4	11 208.5	12 051.0	12 930.4	13 828.5	14 745.4	15 681.0	16 648.5	17 640.4	18 651.0	19 680.4	20 735.4	
3/9	8 704.3	9 520.3	10 381.7	11 293.7	12 245.7	13 221.7	14 221.7	15 257.3	16 325.3	17 417.3	18 533.3	19 681.7	20 865.7	22 073.7	23 305.7	24 566.7	
4/8	7 954.0	8 690.0	9 444.5	10 232.5	11 036.5	11 856.5	12 700.0	13 572.0	14 460.0	15 368.5	16 324.5	17 309.0	18 317.0	19 359.0	20 423.0	21 520.5	
4/9	9 738.2	10 647.1	11 578.2	12 547.1	13 544.9	14 564.9	15 611.6	16 698.2	17 807.1	18 942.2	20 135.6	21 363.9	22 623.9	23 926.1	25 257.2	26 631.1	

TABLE No. 12.—Maximum Shears.

Cooper's 1896 Specification "E 40."
Shears in Thousands of Pounds.
Weights on One Rail.



Panel Points.	Panel Lengths.															
	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
1/5	109.3	114.3	119.2	123.9	128.4	132.8	137.1	141.3	145.5	149.6	153.6	157.7	161.9	166.0	170.1	174.1
1/6	134.5	140.1	145.8	151.3	156.7	162.0	167.2	172.3	177.4	182.4	187.3	192.2	197.3	202.3	207.3	212.3
1/7	158.2	164.4	171.0	177.4	183.7	189 9	196.1	202.2	208.2	214.1	220.0	225.8	231.7	237.7	243.7	249.6
1/8	181.0	188.1	195·4	202.7	210.0	217.2	224.3	231.3	238.3	245.2	252.0	258.8	265.6	272.5	279.4	286.3
1/9	203.1	211.2	219.3	227.6	235.8	243.9	252.0	260.0	267.9	275.7	283.6	291.3	299.1	306.8	314.8	322.6
2/5	63.3	66.1	68.9	71.9	74.8	77.3	80.1	82.9	85.7	88.5	91.2	93.8	96.3	98.8	101.2	103.6
2/6	88.8	92.8	96.6	100.3	104.1	107.6	111.2	114.8	118.4	121.9	125.3	128.7	132.1	135.4	138.7	141.9
2/7	113.6	118.3	122.9	127.4	131.8	136.2	140.7	145.2	149.6	154.0	158.3	162.6	166.8	171.0	175.1	179.2
2/8	137.0	142.5	148.0	153.3	158.6	163.9	169.3	174.7	180.0	185.3	190.5	195.7	200.8	205.9	211.0	216.0
2/9	159.6	166.0	172.4	178.6	184.8	191.0	197.3	203.6	209.9	216.1	222.2	228.3	234.4	240.4	246.4	252.4
3/5	31.0	32.2	33.3	34.3	35.4	36.4	37.5	38.6	39.7	41.0	42.2	43.4	44.7	45.9	47.3	48.6
3/6	50.8	53.3	55.7	58.3	60.7	62.9	65.2	67.4	69.6	71.7	73.9	76.0	78.0	80.0	82.0	84.1
3/7	74.5	78.0	81.3	84.6	87.8	90.9	93.9	96.9	99.8	102.6	105.5	108.2	111.1	114.0	116.8	119.7
3/8	98.0	102.1	106.2	110.2	114.1	118.0	121.8	125.6	129.3	132.9	136.6	140.3	144.1	147.8	151.5	155.1
3/9	120.5	125.5	130.4	135.2	140.0	144.6	149.3	153.9	158.4	163.0	167.6	172.2	176.8	181.4	186.0	190.5
4/5	8.8	9.2	9.6	10.0	10.3	10.7	11.1	11.5	11.8	12.1	12.5	12.8	13.1	13.4	13.7	14.0
4/6	25.1	26.2	27.2	28.0	28.8	29.7	30.5	31.3	32.0	32.8	33.9	34.9	36.1	37.1	38.3	39.4
4/7	42.7	44.4	46.2	48.5	50.7	52.6	54.6	56.6	58.5	60.3	62.2	64.1	65.9	67.6	69.3	71.0
4/8	63.8	66.9	69.8	72.8	75.6	78.4	81.1	83.7	86.3	88.8	91.3	93.8	96.2	98.7	101.0	103.4
4/9	85.8	89.6	93.3	96.9	100.4	103.9	107.3	110.7	114.0	117.3	120.5	123.8	127.0	130.1	133.3	136.4
5/6	6.7	7.1	7.4	7.8	8.1	8.4	8.8	9.1	9.4	9.6	10.0	10.3	10.5	10.8	11.1	11.3
5/7	21.0	21.9	22.8	23.5	24.2	25.0	25.7	26.4	27.0	27.7	28.5	29.2	30.0	30.8	31.9	32.8
5/8	36.9	38.4	39.9	41.7	43.3	45.1	46.7	48.4	50.1	51.8	53.5	55.1	56.7	58.3	59.8	61.3
5/9	56.1	58.6	61.1	63.6	66.2	68.6	71.1	73.4	75.8	78.1	80.3	82.5	84.7	86.9	89.1	91.2
6/7	5.1	5.5	5.8	6.1	6.4	6.8	7.1	7.3	7.6	7.9	8.1	8.4	8.7	8.9	9.1	9.4
6/8	17.9	18.7	19.5	20.2	20.8	21.5	22.1	22.7	23.3	23.9	24.6	25.2	26.0	26.6	27.4	28.1
6/9	32.3	33.7	35.1	36.6	38.1	39.7	41.2	42.6	44.0	45.4	46.7	48.2	49.6	51.0	52.4	53.7
7/8	4.0	4.3	4.7	4.9	5.2	5.5	5.8	6.1	6.3	6.5	6.8	7.0	7.3	7.5	7.7	7.9
7/9	15.4	16.2	16.9	17.5	18.1	18.7	19.3	19.9	20.3	21.0	21.5	22.2	22.8	23.4	24.1	24.7
8/9	3.1	3.4	3.7	4.0	4.3	4.5	4.8	5.1	5.3	5.5	5.7	6.0	6.2	6.4	6.6	6.8

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TABLE No. 13.—VALUES OF "EXCESS LOAD" E, WHICH, WITH 2 000 LBS. PER FOOT ON ONE RAIL, WILL GIVE SAME MOMENTS AND SHEARS AS COOPER'S "E 40" LOADING. UNIT 1 000 LBS.

TRUSSES.														PLATE-GIRDERS.			
MOMENTS.							SHEARS.										
$\frac{n}{N}$	Panel Lengths.						$\frac{n}{N}$	Panel Lengths.						Span.	Moment.	End shear.	Center shear.
	20	23	26	29	32	35		20	23	26	29	32	35				
.....	.....	.....	.....	.....	.....	.....	1/5..	36.7	39.9	41.4	42.0	42.4	42.6	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	1/6..	41.3	43.5	44.6	44.9	44.7	44.8	10	12.5	20.0	15.0
.....	.....	.....	.....	.....	.....	.....	1/7..	44.5	46.0	46.8	46.4	46.3	46.2	15	18.3	25.0	19.1
.....	.....	.....	.....	.....	.....	.....	1/8..	46.8	47.7	48.3	48.2	47.5	47.2	20	21.3	30.0	18.9
.....	.....	.....	.....	.....	.....	.....	1/9..	48.5	49.0	49.5	49.2	48.4	48.0	25	23.8	31.5	19.9
2/5...	31.4	35.5	38.0	38.6	37.9	38.2	2/5..	25.5	27.8	29.5	31.5	32.5	32.7	30	24.7	33.0	20.4
2/6...	37.7	38.2	36.8	37.4	37.6	38.8	2/6..	33.2	35.5	36.8	37.9	38.2	37.9	35	24.8	34.2	22.7
2/7...	37.1	38.0	38.4	38.8	38.8	39.1	2/7..	39.0	40.4	41.0	41.6	41.5	40.9	40	25.6	35.4	24.0
2/8...	38.9	39.5	39.8	39.8	39.5	39.4	2/8..	42.7	43.4	43.7	44.1	43.7	43.0	45	26.2	36.7	24.3
2/9...	40.6	40.8	40.8	40.5	40.0	39.7	2/9..	45.2	45.6	45.7	45.8	45.4	44.5	50	26.1	37.1	24.8
.....	.....	.....	.....	.....	.....	.....	3/5..	17.5	16.8	15.8	15.5	15.8	16.5	55	26.2	37.1	25.3
3/6...	33.2	36.8	38.6	38.7	37.9	36.0	3/6..	21.6	24.6	26.4	27.4	28.0	28.2	60	26.6	37.6	25.8
3/7...	38.4	38.4	39.0	39.0	38.0	36.0	3/7..	30.4	33.1	34.1	34.6	34.4	34.5	65	27.2	38.8	26.3
3/8...	37.0	38.2	39.2	39.2	38.0	36.0	3/8..	36.8	38.3	38.9	38.6	38.6	38.2	70	27.5	40.5	26.4
3/9...	37.6	38.5	39.5	39.3	38.1	35.9	3/9..	40.8	41.8	42.0	41.5	41.2	40.8	75	27.7	42.7	27.1
.....	.....	.....	.....	.....	.....	.....	4/5..	4.0	4.0	3.5	2.5	1.5	0.0	80	28.0	44.2	27.1
.....	.....	.....	.....	.....	.....	.....	4/6..	15.3	15.0	13.5	11.4	12.3	13.2	85	28.3	45.7	28.1
.....	.....	.....	.....	.....	.....	.....	4/7..	19.6	21.2	23.4	24.7	25.8	25.7	90	28.7	47.2	28.4
4/8...	38.9	38.5	36.2	33.0	30.2	27.4	4/8..	27.6	30.6	32.2	32.6	32.4	31.8	95	28.8	48.6	28.5
4/9...	39.2	38.5	36.2	32.9	30.2	27.4	4/9..	34.4	36.4	37.1	37.1	36.6	35.5	100	28.8	50.0	28.8
.....	.....	.....	.....	.....	.....	.....	5/6..	0.2	0.8	0.8	−0.4	−1.0	−2.2	105	29.8	51.1	28.7
.....	.....	.....	.....	.....	.....	.....	5/7..	13.5	13.3	12.0	10.0	9.0	9.8	110	31.3	52.0	28.6
.....	.....	.....	.....	.....	.....	.....	5/8..	18.4	19.2	20.5	22.1	23.2	23.5	115	32.7	52.9	28.7
.....	.....	.....	.....	.....	.....	.....	5/9..	26.2	28.1	30.0	30.7	30.6	30.2	120	33.8	53.7	29.0
.....	.....	.....	.....	.....	.....	.....	6/8..	11.6	11.8	10.4	8.7	8.0	7.5	125	34.9	54.4	29.5
.....	.....	.....	.....	.....	.....	.....	6/9..	16.9	17.8	19.6	20.2	20.8	21.1	.....	.....	.....	.....
.....	.....	.....	.....	.....	.....	.....	7/9..	9.3	9.8	8.9	7.5	6.6	6.2	.....	.....	.....	.....

Mr. Selby practically with the curves of actual values. The equations of the straight lines are as follows:

For end shears	E, for one girder,	$=$	$25 + \frac{1}{4} L.$
" moments	"	"	"	$22 + \frac{1}{10} L.$
" center shears	"	"	"	$20 + \frac{1}{10} L.$

Shears and moments computed from values of E obtained by the foregoing formulas will differ from the actual values by very small percentages on the safe side, except for span lengths of less than 20 ft. The greatest differences are as follows: For moments—10-ft. span, 46.6%, 15-ft. span, 15.5%, 20-ft. span, 6.6%, 100-ft. span, 2.5%. For end shears—10-ft. span, 25%, 15-ft. span, 9.4%, 60-ft. span, 2.5%. For center shears—10-ft. span, 30%, 15-ft. span, 9.1%, 20-ft. span, 14.3%, 25-ft. span, 8%, 30-ft. span, 7.4%, 125-ft. span, 3.3 per cent.

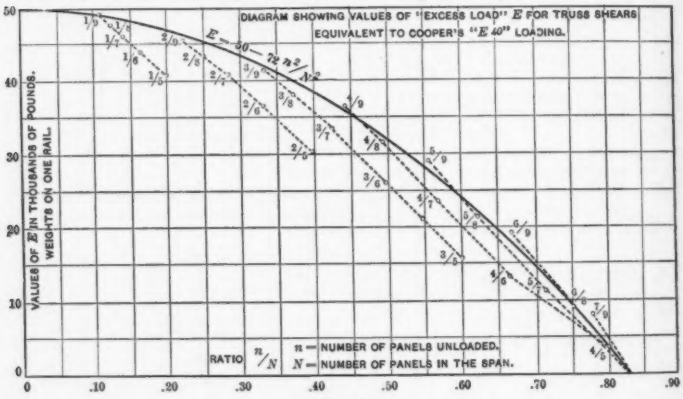

FIG. 9.

By the use of the above formulas for E, the greatest excess over the actual quiescent stress comes, as it should, on the shorter spans, where extra metal in a girder is desirable for the sake of mass. In a specification for actual use the number of formulas could be reduced to two, with advantage, using

$E = 25 + \frac{1}{4} L$ for end shears, and
$E = 22 + \frac{1}{10} L$ for moments and center shears.

The increase in values of center shears would not result in much increase in the weight of metal.

The values of E for truss shears have been plotted on the diagram, Fig. 9, taking the average value at each panel point for the different panel lengths. The abscissas of this diagram represent percentages of span unloaded. The parabola plotted will give values of E which will produce shears practically equal to the actual shears; the greatest

excesses being on the short spans and on counter shears, where they should be. The greatest percentages of difference are as follows: Mr. Selby.

$\frac{1}{5}$	20-ft. panel	7.7%
$\frac{2}{6}$	20 "	12.3%
$\frac{3}{6}$	20 "	8.4%
$\frac{3}{8}$	29 "	8.3%
$\frac{4}{6}$	20 "	3.6%
$\frac{4}{8}$	29 "	7.0%

The equation of the parabola is E, for one truss, $= 50 - 72\frac{n^2}{N^2}$, in which $n =$ number of panels unloaded, and $N =$ whole number of panels. The values of E by this formula, for the different panel ratios, are shown in Table No. 14.

TABLE No. 14.

Panels unloaded.	Whole Number of Panels.				
	5	6	7	8	9
1.	47	48	49	49	49
2.	38	42	44	46	46
3.	24	32	37	40	42
4.	4	18	26	32	36
5.	....	0	13	22	28
6.	....	....	....	10	18
7.	....	....	....	....	6

The use of this formula (or Table No. 14) for E, in a specification, would give shears substantially equal to, and never appreciably less than, those by Cooper's Specification.

The values of E for truss moments do not vary much below 40 for one truss. Using a constant value of 40, the greatest percentages of excess are as follows:

$\frac{2}{5}$	20-ft. panel	6.5%
$\frac{3}{6}$	20 " "	4.4%
$\frac{4}{8}$	32 " "	3.4%
$\frac{4}{8}$	35 " "	4.1%
$\frac{4}{9}$	32 " "	3.1%
$\frac{4}{9}$	35 " "	3.7%

An inspection of the diagrams, Figs. 8 and 9, shows that the method proposed by Mr. Cowles, of using the uniform train-load headed by the maximum floor-beam reaction, will generally give results too small. The panel excess derived from the floor-beam reactions for panels of 20 to 35 ft. varies from 25.6 to 27.5, and averages 26.5 for one truss.

It will be seen that the necessary excess is greater, and usually much greater, than this, except for counter shears and short plate-girders. The writer knows of one bridge now being built under

Mr. Selby. Cooper's Specifications, in which the equivalent loading is taken as 4 000 lbs. per foot, with 50 000 lbs. excess. The specifications of a prominent system call for two classes of loading: Class *A*, 5 000 lbs. per foot, with 50 000 lbs. excess; and Class *B*, 4 000 lbs. per foot, with 40 000 lbs. excess. It is evident from the diagrams that these values of 20 and 25 for *E* are too small to give consistently-designed bridges for a uniform train-load headed by two correspondingly heavy consolidation engines.

It may seem strange, at first glance, that the values of *E* should be smaller for short plate-girders than for longer ones, but it will be found that the value of *E* divided by the span length is greater for the short spans than for the long ones; that is, the "equivalent uniform load," increases as the span decreases.

A specification based on the foregoing formulas, viz., the use of a uniform load of 2 000 lbs. per foot for one truss or girder with a single excess, determined by—

$E = 40$ for truss moments,

$E = 50 - 72\,\frac{n^2}{N^2}$ " " shears,

$E = 25 + \frac{1}{4}\,L$ " plate-girder end shears, and

$E = 22 + \frac{1}{10}\,L$ " " " moments and center shears,

would surely give a consistently proportioned structure, with stresses as nearly equivalent to the actual quiescent ones as it is possible to obtain by simple methods. The uniform load is constant, and the values of *E* are determined by easily memorized formulas, no tables or diagrams being necessary. Mr. Waddell's "equivalent uniform load" method is necessarily cumbersome, because the load for any given span has to be taken from a diagram plotted from computed equivalents, and cannot be memorized. The diagram must be before one when beginning the computation of a span. Mr. Waddell, in his discussion, says: "In these tables the best possible showing was made for both the 'Single-Concentration Method' and the 'Double-Concentration Method.'" It is evident that it is not the best possible showing for the single-excess method to compare results obtained by using a constant value of *E* with those obtained by using a variable value of uniform load computed for the cases compared. To illustrate by an extreme case, if the variable excesses from Table No. 13 were used in a comparison, the showing for the single-excess method would be very good, indeed, as the percentage of error would be zero in every case. It has been shown that the differences from using the foregoing simplified formulas are very small, except for spans less than 20 ft., and that, in all cases, the excess of metal resulting is placed where it will do the most good.

Finally, by introducing the consideration of impact, and, bearing in mind the widely varying ideas as to the proper allowances therefor, it becomes possible to simplify still further the specified loading, and

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to compute at one operation both the quiescent stresses and the impact allowance. Tables Nos. 15, 16 and 17 show the percentage of excess stress from using a constant value of E for all purposes, and a comparison with the allowance for impact specified by the railroad system with which the writer is connected. The formula for impact is $I = \frac{10}{L+10}$, in which L is the span length for plate-girders and for truss moments, and is the length of span loaded for truss shears.

TABLE No. 15.—PLATE-GIRDER SPANS.

	25 ft.	50 ft.	75 ft.	100 ft.	125 ft.
Moments $E = 50$	54%	31 %	22 %	16%	9.5%
" $E = 40$	33%	18.5%	12 %	9%	3.2%
End Shears $E = 50$	33%	15 %	6.2%	0%	0 %
Impact $I = \frac{10}{L+10}$	30%	16.7%	12 %	9%	7.5%

TABLE No. 16.—TRUSS-CENTER MOMENTS, 20-FT. PANELS.

	5 panels.	6 panels.	7 panels.	8 panels.	9 panels.
$E = 50$	14%	10 %	6.5%	5.6%	5 %
Impact $I = \frac{10}{L+10}$	9%	7.7%	6.7%	6 %	5.3%

TABLE No. 17.—TRUSS SHEARS. 9 PANELS AT 20 FT.

Panel points	1.	2.	3.	4.	5.	6.	7.
$E = 50$	0.6%	2.3%	5.0%	10%	19%	34%	58.5%
Impact $I = \frac{10}{L+10}$	6.0%	6.7%	7.7%	9%	11%	14%	20.0%

Ideas vary so much as to the correct allowance for impact, that the above formulas will answer for comparison as well as any, and the correspondence is close enough to make it plain that the use of a constant single-excess load for all spans, except plate-girder spans under 20 ft., comes very near to giving, with one operation, the desired result, namely, the actual stress plus the impact.

The writer would, therefore, favor, for a standard railroad bridge specification, the use of a uniform load of 4 000 lbs. per foot of track, together with a single excess of 100 000 lbs., placed at the most effective point, with no other allowance for impact for all spans over about 20 ft. For spans under 20 ft. stresses would be determined by a series of the heaviest passenger-axle loads with an impact allowance of $I = \frac{10}{L+10}$.

Mr. Prichard. HENRY S. PRICHARD, M. Am. Soc. C. E. (by letter).—If the term Railroad Bridge is understood to include the track, then the loads to which railroad bridges are subjected are concentrated wheel-loads. These loads are somewhat distributed over the remainder of the structure by the track, which slightly lessens the stresses in the connections of stringers to floor beams and reduces very considerably the pressures on ties and on local points in the flanges of stringers, but, except in these regards, the stresses produced are the same as if the loads were concentrated directly on the flanges of the stringers.

The most accurate method of specifying the loading to which a bridge is subjected is, of course, that of specifying the actual loads. For present conditions, the critical loadings are approximately knowable, therefore, for present conditions, the most accurate method of specifying the loading is to specify wheel concentrations. If, in a bridge designed for present conditions, the loads increase, but still maintain the same relations between the various wheel concentrations of both the engines and the cars, then the live-load stresses will be increased in the same proportion, and if the bridge was originally well designed for the original wheel concentrations, the distribution of the material will remain the most economical for the increased loading. Even if the relations between the wheel concentrations should vary greatly in the future, in a manner not intentionally provided for, there is no reason to suppose that any method other than that of specifying wheel concentrations would be more accurate as regards these future conditions, except by mere chance. From the foregoing facts, it is evident that, as far as accuracy is concerned, there is no reason why wheel concentrations should not continue to be specified.

Mr. Scofield and Mr. Snow have pointed out, what few if any will deny, that to those, including many bridge buyers, who are not expert in bridge design, a definite engine-loading is more intelligible than methods of supposed equivalents.

In fact, the method of specifying wheel concentrations is the only one which those who are not experts can use understandingly to give a close equivalent for the traffic conditions they anticipate, unless they obtain the assistance of an expert. Even for experts, the easiest method of specifying loads which satisfy their ideas regarding the traffic conditions, is to specify the engine wheel-concentrations.

This is shown by the fact, well illustrated in the discussion, that nearly all who have proposed methods other than by wheel-concentrations, have arrived at the loads they specify by first deciding on certain engine wheel-concentrations and then, with more or less labor (generally considerable), determining on supposedly close equivalents.

From what has been stated, it follows that, as far as facility in

writing specifications is concerned, there is no reason why the method of specifying wheel concentrations should not be continued. Mr. Prichard.

The questions of accuracy and facility of specifying the loads being decided in favor of the method of engine wheel-concentrations, the question of the wisdom of abandoning its use depends on the value of the advantage as to accuracy, compared with the increased cost, if any, by which this advantage is obtained. To those who are not familiar with the short ways which have been devised for determining the stresses from wheel concentrations, or who, from lack of practice, cannot use them with facility, the cost of making the calculations probably seems much greater than it really need be. For stringers, floor beams, hangers, plate-girders and single-intersection trusses, resting on two supports, the calculation of the stresses from concentrated wheel-loads by proficient specialists is a very simple matter; in fact, it takes less time than by using some of the other systems which have been proposed, and the cost is insignificant, as compared with the total cost of the bridge. Such cases include the great majority of railway bridge work, and for them there would seem to be little advantage in substituting for this method an approximately equivalent one, even if a satisfactory one is obtainable. Be this as it may, it is certain that before the method of wheel concentrations can be abandoned, some other method or methods must be selected to take its place. From present appearances it would be a perfectly safe proposition for the advocates of this method to agree to abandon its use when its opponents unite in recommending some other to take its place.

For the trusses of long spans it is not especially difficult to obtain other methods which give results agreeing closely with those obtained from any system of wheel concentrations of usual type, and a fair agreement can be obtained for trusses of spans of moderate length, but for short spans and for floor systems the writer knows of no other method which gives such close approximations in all cases. For multiple-intersection trusses, for continuous girders, and for some other special cases there is much more reason for adopting some approximately equivalent method, than there is in the cases previously discussed, because, for such cases, the use of the method of wheel-loads is often quite complicated, and in some instances it is extremely so. For such cases, even when wheel-loads are specified, approximately equivalent methods are frequently allowed and used.

There is one phase of the use of the method of wheel concentrations which requires notice, because misapprehension regarding it has resulted in throwing not a little discredit upon it. The fact referred to is, that this method has a superficial appearance of much nearer approach to accuracy than it really possesses, owing to the uncertainty as to future traffic conditions, the impossibility of making

Mr. Prichard. accurate allowance for the dynamic effect of the load, and other pertinent considerations. This fact has made the method of wheel concentrations a butt for ridicule. In as far as this ridicule is the means of disabusing the minds of those who would otherwise be deceived by the seeming accuracy of the method it is commendable, but to the considerable extent by which it discredits the real advantage as to the accuracy of this method, as compared with others, it is harmful. The thoughtful advocates of the use of the method of wheel concentrations advocate its use in spite of, and not in consequence of, its seeming accuracy, because, notwithstanding the fact that it is really far from accurate, it approaches nearer to accuracy than any other method. If those who believe that this advantage as to accuracy is worth what it costs in time and trouble should refrain from using this method simply because it has a superficial appearance of accuracy which is deceptive, they would be allowing their judgment to be influenced by purely sentimental considerations.

While the fact that it is not possible to obtain the stresses from the live load accurately is not a sufficient warrant for abandoning the use of the method of wheel concentrations, it affords an excellent reason for departing from the exact weights and spacings of the wheel concentrations and substituting therefor some simplified typical engine.

Mr. Wing. CHARLES B. WING, Assoc. M. Am. Soc. C. E. (by letter).—With our present limited knowledge of the stresses in bridge superstructures, due to rapidly moving train loads, any refinement of calculation of stresses from assumed loads is not justified, if the cost of the structure is thereby increased. To quote an axiom familiar to bridge engineers: "Stresses should not be figured to half a pound when you guess at the load."

In discussing the advisability of discontinuing the use of the method of wheel concentrations in determining the stresses in railway bridges, certain conditions of present practice should be considered carefully. The loading now in common use is that of wheel concentrations, and there seems to be no agreement among engineers as to the best substitute for this loading.

All the substitutes generally advocated involve the use of wheel concentrations in their computation. If the method of wheel concentrations is specified by purchasers, competition among bridge manufacturers compels the use of the actual wheel concentrations specified in computing stresses. Railway bridge engineers are in the habit of testing the strength of their existing structures by actual engines. They have moment diagrams calculated for the various types of engines in use on their system, and, naturally, prefer the method of wheel concentrations for calculating stresses. Any proposed change in the present method of specifying the load must meet with the approval of railway bridge engineers if it is to be adopted for general use.

Mr. Wing.

The principal objection urged against the use of the method of wheel concentrations is the greater labor involved in calculating stresses thereby.

This extra labor is so slight that its cost cannot be considered a sufficient reason for rejecting the method. Few engineers connected with bridge shops have admitted that the labor is great when a moment diagram has once been computed. Such a diagram is necessary for computing any of the substitute methods suggested.

The writer, therefore, suggests, as a solution of the question under discussion, that railway engineers, in writing specifications, include a moment diagram of the wheel concentrations as a part of the specification.

This will reduce materially the labor of the estimating department of every bridge manufactory, no matter what system of loading is finally adopted for computing stresses.

Such moment diagrams are part of the office equipment of every railway bridge engineering department, and including them in a specification involves no extra labor on the part of the bridge purchaser.

Mr. Marburg.

EDGAR MARBURG, M. Am. Soc. C. E. (by letter).—The current discussion of the faults and merits of the method of wheel concentrations seems to indicate that the settlement of this long-mooted question is as remote to-day as it has ever been. The so-called "equivalent-load systems" bid fair to rival, in variety and in multiplicity, the wheel systems which they are designed to supplant. The only hope of an approach to some agreement would seem to lie in a report on the subject by a Special Committee. The writer shares the view expressed by Mr. Thacher, that unification will never be achieved through the efforts of any single individual. If, on the other hand, a Committee would formulate a very limited number (perhaps not exceeding three or four) of "basic" wheel systems, in which the loads may be varied throughout by percentages chosen at pleasure, it may be reasonably expected that they would come into very general use. This excellent method of variation by percentages has already been adopted by Mr. Cooper.

It would seem that the following general method of procedure should meet with the approval of the most ardent partisans of both factions. The opponents of wheel systems could then center their efforts upon the few standard systems which might be chosen, and derive the necessary "constants" and "variables" for the particular scheme in view, with any desired degree of elaboration. From the standpoint of the bridge companies, and expert computers generally, the objection is not so much to wheel systems as such, but to the endless and needless variety in which they appear.

In the opinion of the writer, the arguments against the method of wheel concentrations are seldom fairly presented. Without reference

Mr. Marburg. to the speculative element of increased loading in the future, there are admittedly a dozen or more important reasons in support of the well-known and universally admitted fact that the computed stresses, based on wheel systems, represent at best only fair approximations to actual conditions. There is, in fact, no adequate ground for assuming that such stresses accord any better with the maximum stresses in service than those found by any one of the "equivalent" methods, provided the two sets of computed stresses agree within reasonable limits. This latter qualification is, however, the all-important one. It may be properly claimed that no simple system of "equivalent" loading has been proposed which is sufficiently comprehensive and flexible to meet satisfactorily the many "special cases" of common occurrence in miscellaneous practice. The maximum reactions from adjacent spans of unequal lengths may be cited as a single example. This applies to viaduct posts, to end floor-beams in skew bridges, and to the suspenders for the latter. Other examples may be found in swing bridges, cantilevers, turn-tables, and unusual constructions and combinations generally. However artificial the wheel system may be, it has the great merit of consistency under all conceivable circumstances.

The comparisons between wheel systems and "equivalent" systems are usually restricted to trusses with horizontal chords, although the latter are the exception rather than the rule for spans exceeding 200 ft. While the ratio of the chord stresses in such comparisons remains, of course, unaffected, by the inclination of the chord, that of the web stresses is greatly modified. The writer has occasionally made such comparisons, but not in sufficient variety and number to warrant definite deductions.

From the standpoint of the railroad companies, the writer would not regard it expedient, or, in fact, safe, to abandon the wheel system. A prescribed engine loading should always be available for special conditions and as a general criterion in doubtful cases. Tables or diagrams could readily be appended to the specifications, giving (*a*) the maximum bending moments and shears for girders; (*b*) the shears at the center and quarter points; (*c*) the floor-beam reactions for equal panels; and (*d*) the equivalent uniform-loads for chord stresses. The latter the writer would determine from the end shear, for an assumed average panel length, rather than from the bending moment at the quarter point. The results are slightly greater, but involve no errors (differences) on the side of danger in the end panels. Compared with the chord stresses from the wheel system itself, the increase is seldom greater than 4% (usually, then, in only a single panel), and very seldom or never as high as 5 per cent. The writer has applied this method for several years in practice to a great variety of wheel loads as an approximate check on the more refined methods, with results as stated.

Mr. Marburg.

The writer prefers this method of equivalent loads also for web stresses in trusses with parallel chords. It obviates the differences on the side of danger incidental to the quarter-moment method,* provided the forward half-panel loads be neglected, as should be done in all cases.

For the web stresses in trusses with inclined chords, the writer is not prepared to endorse any of the so-called equivalent methods, nor is he aware that this phase of the subject has ever been sufficiently investigated.

The writer prefers the above-mentioned equivalent uniform-load method to the methods of added single and double concentrations, not because it accords better with the stresses from wheel loads (in fact, the contrary is true), but on account of its greater simplicity. Besides, where its use is restricted as outlined above, the errors are all on the side of safety.

The possibilities of the added-concentration methods in the way of so-called accuracy cannot be revealed adequately by the method chosen by Mr. Waddell in compiling his tables of comparative values. Either the concentrated load should be constant and the uniform load variable, as suggested some years ago,† or *vice versa*, as here proposed by Mr. Selby.

* These differences may be much greater than those exhibited in Mr. Waddell's tables in the present discussion. See Johnson's "Modern Framed Structures," pp. 107, 108.

† *Ibid*, p. 102.